**Name……………………………………………Stream………Teacher………………**

**DEPARTMENT OF MATHEMATICS**

S.6 PURE MATHEMATICS–2020

**PAPER 1 WEEK 1**

3 HOURS

- *Answer all the eight questions in section A and any five from section B.*
- *Any additional question(s) answered will not be marked.*

## SECTION A: (40 MARKS)

1. Solve the equation $^{p}(6x + 1) - {}^{p}(2x - 4) = 3$. (*05 marks*)

2. Solve $\cos\theta = \sin(\theta + 30^0)$ in the range $0^0 \leq \theta < 360^0$. (*05 marks*)

3. Show that $\frac{1}{\sqrt{4-x}} = \frac{1}{2}\left(1 - \frac{x}{4}\right)^{\frac{-1}{2}}$. Write down the first three terms in the binomial expansion of $\left(1 - \frac{x}{4}\right)^{\frac{-1}{2}}$ in ascending powers of $x$. (*05 marks*)

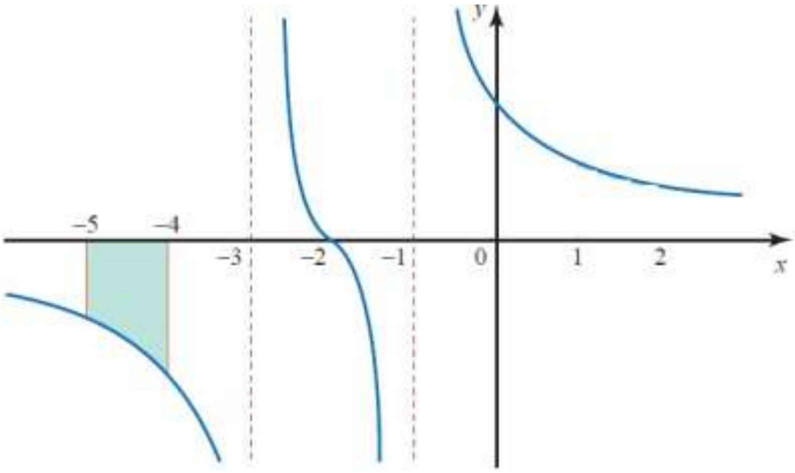
4. Evaluate $\int_0^{\frac{\pi}{8}} \frac{e^{\tan 2x}}{\cos^2 2x}dx$. (*05 marks*)

5. Find the shortest distance from point $P(11,-5,-3)$ to the line $l$ with equation $\mathbf{r} = \begin{pmatrix} 1 \\ 5 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ 1 \\ 4 \end{pmatrix}$. (*05 marks*)

6. An open box with a square base has a total surface area of 300 cm$^2$. Find the greatest possible volume. (*05 marks*)

7. Find the area of the largest square contained within the circle $x^2 + y^2 - 2x + 4y + 1 = 0$. (*05 marks*)

8. The graph of $y = \dfrac{x+2}{x^2+4x+3}$ is shown below.




Find the area of the shaded region. (*05 marks*)

## SECTION B: (60 MARKS)

9. (a) Prove that $\dfrac{d}{dx}(uv) = v\dfrac{du}{dx} + u\dfrac{dv}{dx}$.

(b) Differentiate the following.

(i) $(x-1)^{\frac{1}{3}}(x-2)^3$.

(ii) $\dfrac{2x^2-3x}{(x+4)^2}$. (*12 marks*)

10. (a) Find the equation of the circle passing through the points $A(3,2)$, $B(-1,0)$ and $C(5,-2)$. (06 marks)

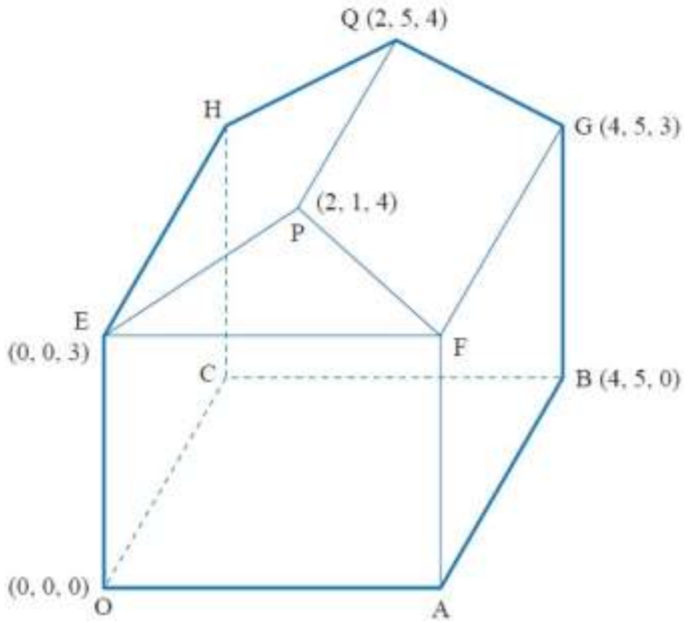
(b) Show that the locus of the point $P$ with co−ordinates $(1 + 2\cos\theta, 2 + 2\sin\theta)$ is a circle and find its radius and centre.

(06 marks)

11. (a) Find $\frac{d^2y}{dx^2}$ when $y = \sin^{-1} x - x\sqrt{(1 - x^2)}$, expressing your answer as simple as possible.

(b) Use Maclaurin's theorem to express $\ln\sqrt{\frac{1+x}{1-x}}$ as a power series up to the term in $x^3$. (*12 marks*)

12. The diagram shows an extension to a house. Its base and walls are rectangular and the end of its roof, *EPF*, is sloping, as illustrated.




(a) Write down the co–ordinates of *A* and *F*.

(b) Find, using vector methods, the angle *EPF*.

(c) The owner decorates the room with two streamers which are pulled taut. One goes from *O* to *G*, the other from *A* to *H*. She says that they touch each other and that they are perpendicular to each other. Is she right? (*12 marks*)

13. Bacteria in a culture increase at a rate proportional to the number of bacteria present. If the number increases from 3000 to 4000 in one hour,

(a) how many bacteria will be present after $2\frac{1}{2}$ hours. (*09 marks*)

(b) how long will it take for the number of bacteria in the culture to become 6000? (*03 marks*)

14. (a) Solve the simulateous equations

$$(x + 3)(y + 3) = 10 \text{ and } (x + 3)(x + y) = 2.$$

(*05 marks*)

(b) Use the substitution $y = x + \frac{2}{x}$ to solve
$x^4 - 5x^3 + 10x^2 - 10x + 4 = 0.$ (*07 marks*)

15. (a) Use the substitution $t = \tan\theta$ to solve $\sin 2\theta + 2\cos 2\theta = 1$ for $0 < \theta \leq 2\pi$. (*05 marks*)

(b) The equation $1+\sin^2 \theta^0 = a\cos 2\theta^0$ has a root of 30. Find the value of $a$ and all the roots in the range 0 to 360. (*07 marks*)

16. (a) Prove by induction that $\sum_{r=1}^{n} r(r+1) = \frac{1}{3}n(n+1)(n+2)$. (*05 marks*)

(b) Aisha opens an account with a saving scheme which offers a 12.9% compound interest per annum. The scheme does not allow any withdrawal until after a period of 5 years. A customer is required to deposit a half the amount of money he/she opens the account with every beginning of other years for a period of 4 years. If Aisha started with shs 600,000, calculate how much;

(i) money she will earn from the scheme after 5 years.

(ii) interest she earns from the saving scheme. (*07 marks*)

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