P425/1
PURE MATHEMATICS
PAPER 1
July/August 2017
3hrs

RESOURCEFUL MOCK EXAMINATIONS, 2017

Uganda Advanced Certificate of Education

PURE MATHEMATICS

(P425/1)

TIME: 3HOURS

INSTRUCTIONS TO CANDIDATES

- ✓ *Attempt all the questions in section A and five from section B.*
- ✓ *Working must be shown clearly*
- ✓ *Silent non programmable calculator may be used.*
- ✓ *Any additional question(s) answered will not be marked.*

SECTION A

1. Prove that

$$\tan^{-1} x + cot^{-1}x = \frac{\pi}{2}$$

2. The first term of an AP and G.P are each $\frac{2}{3}$ their common difference and common ratio are x and the sum of their first 3 terms is equal. Find the possible values of x.

3. $\int 2^{\sqrt{3x-1}}\,dx.$

4. Solve$3sin\left(2x + {}^{\pi}/_{6}\right) - \cos\left(2x + {}^{\pi}/_{6}\right) = 2$

5. Find the equation of the normal to the curve $\frac{y}{x+siny} = 3$at the point where $y = \pi$.

6. Show that when the quadratic expression.

$x^2 + bx + c = 0 \; and \; x^2 + px + q = 0$have a common root then

$$(c - q)2 = (b - p)(pc - bq)$$

7. Given that

P $= \log_2 3$ $and\ q = \log_4 5$, show that $\log_{45} 2 = \frac{1}{2(p+q)}$

8. Use the substitution.

$$y = x + \frac{1}{x}\ to\ solve\ the\ equation\ 2x^4 - 9x^3 + 14x^2 - 9x + 2 = 0$$

SECTION B

9. Describe the locus of the complex number z which moves in the argand diagram.
$Arg\left(\frac{z-3}{z-2i}\right) = \frac{\pi}{2}$
b) Find the fourth roots of -16i

10. If A, B and C are angles of a triangle prove that
$sin^2 A + sin^2 B + sin^2 C = 2 + 2cosAcosBcosC$
b) By expressing 6cos$^2\theta + 8sin\theta cos\theta$ in the form $Rcos(2\theta - 2)$. Find the maximum and minimum value of $6cos^2\theta + 8sin\theta cos\theta = 4$

11. The curve with the equation $y = \frac{ax+b}{x(x+2)}$ where a and b are constants has a turning point at (1, -2). Find the values of a and b.
Find the equation of all the asymptotes.
Sketch the curve.

12. Differentiate
$y = 2x^{cosx}$
$y = \frac{e^{sinx}}{tanx}$
b) Prove that $\int_1^3 \left(\frac{3-x}{x-1}\right)^{\frac{1}{2}} dx = \pi$. Use the substitution $x = 3sin^2\theta + cos^2\theta$.
c) The displacement of a particle at time t is x measured from a fixed point and $\frac{dx}{dt} = \frac{C\,(e^{2act}-1)}{e^{2act}+1}$, prove that $x = \frac{C\,(e^{2act}-1)}{e^{2act}+1}$, if x = 3 when t = 1 and x = $\frac{75}{17}$, prove that c = 5

13. Show that the lines
$r = 2i - 3j + 4k + \lambda(3i - 2j + k) and\ r = i + 3j + k + \mu(-i - 2j + k)$intersect. Find the point of intersection.
b) OAB is a triangle with OA = $\underset{\sim}{a}$, $OB = \underset{\sim}{b}$, c is amidpoint of OB, D is the midpoint of AB and E is amidpoint of OA. OD and AC interest at F. if AF = hAC and OF = KOD. Find the values of h & k. show that B, F & E are collinear.

14. a) Solve $\frac{dy}{dx} + 2y\tan x = \cos^2 x$

$$y(0) = 2$$

b) A radioactive substance disintegrates at a rate proportional to it's mass one half of the given mass of a substance distergrates 136 days, calculate the time required for $\frac{5}{8}$ of a substance to disintergrate. If the original mass of a substance was 100gm. Calculate the mass after 34 days.

15. Find the equation of the tangents to the curve at $y = x^3$ at (t, t^3) prove that this tangent meets the curve again at $Q(-2t - 8t^3)$. Find the locus of the midpoint of PQ.

b) Given that $y = mx + c$ is a tangent to the circle $(x - a)^2 + (y - b)^2 = r^2$. Show that $(1 + m^2)r^2 = (c-b+am)^2$.

16. a) $\int_1^2 \frac{8x+6}{(2x-1)^2\,(x+2)^2}$

b) $\int_0^{\pi/2} \frac{1}{2+\cos^2 x}\,dx$

END