P425/1 PURE MATHEMATICS Paper 1 Jul. / Aug. 2016 3 hours



"Together for Mathematics"

SECONDARY MATHEMATICS TEACHERS' ASSOCIATION

SMATA JOINT MOCK EXAMINATIONS 2016 Uganda Advanced Certificate of Education PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

Answer all the *eight* questions in Section **A** and **five** questions from Section **B**.

Any additional question(s) answered will **not** be marked.

All working **must** be shown clearly.

Begin each answer on a **fresh** sheet of paper.

Graph paper is provided.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

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SECTION A: (40 MARKS)

Answer **all** questions in this Section.

1. Solve the simultaneous equations

$$x y = 2$$

$$2\log(x-1) = \log y$$
(05 marks)

2. Differentiate
$$y = \frac{x-2}{\sqrt{\left(1-x^2\right)}}$$
 with respect to x. (05 marks)

3. Without using tables or calculators, show that $\tan^2 22.5^\circ = 3 - 2\sqrt{2}$. (05 marks)

4. Find the gradients of the two tangents from the point (3, -2) to the circle $x^2 + y^2 = 4$. (05 marks)

5. Evaluate
$$\int_0^{\frac{\pi}{2}} \frac{\sin x}{3 + 5\cos x} dx$$
 (05 marks)

6. The points A and B have position vectors $4\mathbf{i} + 3\mathbf{j}$ and $\mathbf{i} + t\mathbf{j}$. Determine the values of t such that the angle $A\hat{O}B = \cos^{-1}\frac{2}{\sqrt{5}}$, where O is the origin. (05 marks)

- A hemispherical bowl of internal radius 15 cm contains water to a depth of 7 cm, find the volume of the water in the bowl correct to 1 decimal place. (05 marks)
- 8. Compute the sum of four- digit numbers formed with the four digits2, 5, 3, 8 if each digit is used only once in each arrangement.

(05 marks)

SECTION B: (60 MARKS)

Answer **only** five questions. All questions carry equal marks.

- 9. (a) Show that the curve $x = 5 6y + y^2$ represents a parabola and find the directrix. (05 marks)
 - (b) (i) Find the equation of the chord through the points $P(ap^2, 2ap)$ and $Q(aq^2, 2aq)$ of the parabola $y^2 = 4ax$.
 - (ii) Show that the chord in (b) (i) cuts the directrix where $y = \frac{2a(pq-1)}{p+q}$ (07 marks)

10. (a) The roots of the equation $2x^2 - 3x + 5 = 0$ are α and β . Find the equation whose roots are $\frac{\alpha}{\beta-2}$ and $\frac{\beta}{\alpha-2}$. (06 marks)

(b) Solve the equation
$$\sqrt{\frac{x-1}{3x+2}} + 2\sqrt{\frac{3x+2}{x-1}} = 3.$$
 (06 marks)

11. (a) Given that $x = \sec A - \tan A$, prove that $\tan \frac{A}{2} = \frac{1-x}{1+x}$. (05 marks) (b) Solve the equation $\sin t \cos 3t + \sin 3t \cos t = 0.8$ for $0 \le t \le 2\pi$.

(05 marks)

12. Given that
$$y = \sin(2\sin^{-1}x)$$
, prove that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y = 0$. Hence,

by Maclaurin's theorem, expand y as far as the term in x^3 .

(12 marks)

- 13. The position vectors of points P and Q are $2\mathbf{i} 3\mathbf{j} + 4\mathbf{k}$ and $3\mathbf{i} - 7\mathbf{j} + 12\mathbf{k}$ respectively.
 - (a) Determine the length of PQ. (03 marks)
 - (b) Given that the line PQ meets the plane 4x + 5y 2z = 5 at the

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point T. Find the:

- (i) co-ordinates of T,
- (ii) angle between line PQ and the plane. (09 marks)

14. (a) Express
$$y = \frac{x^4 + 2x}{(x-1)(x^2+1)}$$
 into partial fractions. Turn Over 07 marks)

(b) Evaluate $\int_{2}^{4} y \, dx$. (05 marks)

15. (a) Show that
$$2 - 3i$$
 is a root to the equation.
 $z^4 - 5z^3 + 18z^2 - 17z + 13 = 0$.

Hence find the other roots of the equation. (06 marks)

(b) Using De Moivre's theorem, find the cube roots of -4 + 6i.

(06 marks)

16. (a) Solve the differential equation
$$\frac{dr}{d\theta} + 2r \tan \theta = \sec^2 \theta$$
. (05 marks)

(b) The tangent at any point P(x, y) on the curve, cuts the x-axis at A and the y-axis at B. Given that $2\mathbf{AP} = \mathbf{PB}$ and that the curve passes through the point (1, 1), find the equation of the curve. (07 marks)

END