P425/1
PURE MATHEMATICS
Paper 1
Jul. / Aug. 2016
3 hours
"Together for Mathematics".

## SECONDARY MATHEMATICS TEACHERS' ASSOCIATION

SMATA JOINT MOCK EXAMINATIONS 2016
Uganda Advanced Certificate of Education PURE MATHEMATICS

## Paper 1

## 3 hours

## INSTRUCTIONS TO CANDIDATES:

Answer all the eight questions in Section $\mathbf{A}$ and five questions from Section B.
Any additional question(s) answered will not be marked.
All working must be shown clearly.
Begin each answer on a fresh sheet of paper.
Graph paper is provided.
Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

## SECTION A: (40 MARKS)

Answer all questions in this Section.

1. Solve the simultaneous equations

$$
\begin{gathered}
x y=2 \\
2 \log (x-1)=\log y
\end{gathered}
$$

(05 marks)
2. Differentiate $y=\frac{x-2}{\sqrt{\left(1-x^{2}\right)}}$ with respect to x .
(05 marks)
3. Without using tables or calculators, show that $\tan ^{2} 22.5^{0}=3-2 \sqrt{2}$.
(05 marks)
4. Find the gradients of the two tangents from the point $(3,-2)$ to the circle $x^{2}+y^{2}=4$. (05 marks)
5. Evaluate $\int_{0}^{\frac{\pi}{2}} \frac{\sin x}{3+5 \cos x} d x$. (05 marks)
6. The points A and B have position vectors $4 \mathbf{i}+3 \mathbf{j}$ and $\mathbf{i}+t \mathbf{j}$. Determine the values of t such that the angle $A \hat{O} B=\cos ^{-1} \frac{2}{\sqrt{5}}$, where O is the origin.
(05 marks)
7. A hemispherical bowl of internal radius 15 cm contains water to a depth of 7 cm , find the volume of the water in the bowl correct to 1 decimal place.
8. Compute the sum of four- digit numbers formed with the four digits $2,5,3,8$ if each digit is used only once in each arrangement.
(05 marks)

## SECTION B: (60 MARKS)

Answer only five questions. All questions carry equal marks.
9. (a) Show that the curve $x=5-6 y+y^{2}$ represents a parabola and find the directrix.
(b) (i) Find the equation of the chord through the points

$$
P\left(a p^{2}, 2 a p\right) \text { and } Q\left(a q^{2}, 2 a q\right) \text { of the parabola } y^{2}=4 a x .
$$

(ii) Show that the chord in (b) (i) cuts the directrix where

$$
y=\frac{2 a(p q-1)}{p+q}
$$

10. (a) The roots of the equation $2 x^{2}-3 x+5=0$ are $\alpha$ and $\beta$. Find the equation whose roots are $\frac{\alpha}{\beta-2}$ and $\frac{\beta}{\alpha-2}$.
(b) Solve the equation $\sqrt{\frac{x-1}{3 x+2}}+2 \sqrt{\frac{3 x+2}{x-1}}=3$.
(06 marks)
11. (a) Given that $x=\sec A-\tan A$, prove that $\tan \frac{A}{2}=\frac{1-x}{1+x}$. ( 05 marks)
(b) Solve the equation $\sin t \cos 3 t+\sin 3 t \cos t=0.8$ for $0 \leq t \leq 2 \pi$.
(05 marks)
12. Given that $y=\sin \left(2 \sin ^{-1} x\right)$, prove that $\left(1-x^{2}\right) \frac{d^{2} y}{d x^{2}}-x \frac{d y}{d x}+4 y=0$. Hence, by Maclaurin's theorem, expand $y$ as far as the term in $x^{3}$.
(12 marks)
13. The position vectors of points $P$ and $Q$ are $2 \mathbf{i}-3 \mathbf{j}+4 \mathbf{k}$ and $3 \mathbf{i}-7 \mathbf{j}+12 \mathbf{k}$ respectively.
(a) Determine the length of $\mathbf{P Q}$.
(b) Given that the line PQ meets the plane $4 x+5 y-2 z=5$ at the
point T. Find the:
(i) co-ordinates of T ,
(ii) angle between line PQ and the plane.
14. (a) Express $y=\frac{x^{4}+2 x}{(x-1)\left(x^{2}+1\right)}$ into partial fractions.
(b) Evaluate $\int_{2}^{4} y d x$.
(05 marks)
15. (a) Show that $2-3 i$ is a root to the equation.

$$
z^{4}-5 z^{3}+18 z^{2}-17 z+13=0 .
$$

Hence find the other roots of the equation. (06 marks)
(b) Using De Moivre's theorem, find the cube roots of $-4+6 i$.
(06 marks)
16. (a) Solve the differential equation $\frac{d r}{d \theta}+2 r \tan \theta=\sec ^{2} \theta$. ( 05 marks)
(b) The tangent at any point $P(x, y)$ on the curve, cuts the x -axis at A and the y -axis at B . Given that $2 \mathbf{A P}=\mathbf{P B}$ and that the curve passes through the point $(1,1)$, find the equation of the curve.
(07 marks)
END

