CHAPTER 4: NEWTON'S LAWS OF MOTION

LAW I: Everybody continues in its state of rest or uniform motion in **a straight line** unless acted upon by an external force.

This is sometimes called the law of **Inertia**

Definition

Inertia is the reluctance of a body to start moving once it's at rest or to stop moving if its already in motion.

Explain why a passenger jerks forward when a fast moving car is suddenly stopped.

Passengers jerk forward because of inertia. When the car is suddenly stopped, the passenger tends to continue in uniform motion in a straight line because the force that acts on the car does not act on the passenger

LAW II: The rate of change of momentum of a body is directly proportional to the applied force and takes place in the direction of the force.

Consider a mass m moving with velocity u. If the mass is acted on by a force F and its velocity changes to v;

By Newton's law of motion

$F \ \alpha \ \frac{mv-mu}{t} = \frac{k(mv-mu)}{t} = km\frac{(v-u)}{t} = k\,m\,a$

Since $a = \frac{v-u}{t}$

When $F = 1N$, $m = 1kg$ and $a = 1ms^{-2}$

$1 = kx1x1$

$k = 1$

$\boxed{F = m\,a}$

Note: F must be the resultant force

LAW III: To every action there is an equal but opposite reactions.

$F_1 = -F_2$

Example of 3rd law of motion

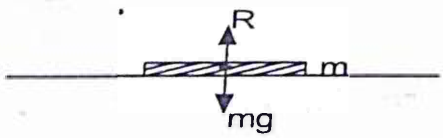
- A gun moves backwards on firing it.
- A ball bounces on hitting the ground.

Rocket engine propulsion

Fuel is burnt in the combustion chamber and exhaust gases are expelled at a high velocity. This leads to a large backwardmomentum. From conservation of momentum an equal forward momentum is gained by the rocket, due to continuous combustion of fuel there is a change in the forward momentum which leads to the thrust hence maintaining the motion of the rocket

4.1.0: IDENTIFICATION OF FORCES AND THE APPLICATION OF NEWTON'S LAWS

1. Consider a body of mass **m** placed on either a stationary platform or a platform moving at a constant velocity

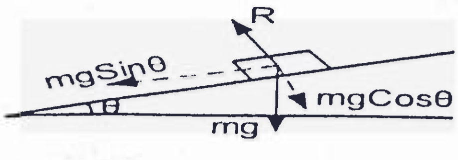


R is normal reaction

Mg is gravitational pull [weight]

$R = mg$ since (a=0) constant velocity

2. Mass m placed on a smooth inclined plane of angle of inclination θ



R = mgCosθ

NB:

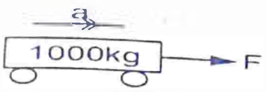
- All objects placed on, or moving on an inclined plane experience a force $mgsin\theta$ **down** the plane. [It doesn't matter what direction the body is moving]
- If the plane is **rough** the body experiences a frictional force whose direction is opposite to the direction of motion.

Examples:

1. A car of mass 1000kg is accelerating at $2ms^{-2}$.
   (i) What resultant force acts on the car?.
   (ii) If the resistance to the motion is 1000N, what force is due to the engine?

Solution

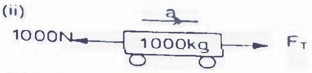
(i)



$F = ma = 1000x2 = 2000N$

Resultant force is 2000N

(ii)

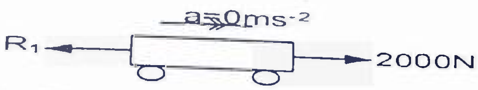


The resistance force should act in opposite direction to the force due to the engine

$F_T - 1000 = ma$
$F_T - 1000 = 1000x2$
$F_T = 3000N$

2. A car moves along a level road at a constant velocity of 22m/s. If its engine is exerting a forward force of 2000N, what resistance is the car experiencing

Solution



Using $F = ma$

$2000 - R_1 = ma$

But a = 0 since it moves with constant velocity

$2000 - R_1 = 0$

$R_1 = 2000N$

3. Two blocks A and B conected as shown below on a horizontal friction less floor and pulled to the right with an acceleration of $2ms^{-2}$ by a force P, if m_1 = 50kg and m_2 = 10kg. what are the values of T and P

Solution

Using $F = ma$

For m_1: $P - T = 50x2 = 100$.......[1]

For m_2: $T = 10x2 = 20N$

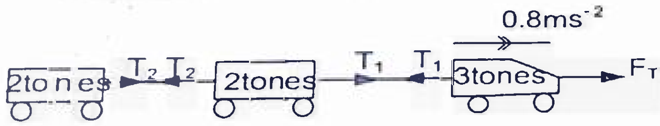
Put into equation (1) $P - T = 100$

$P - 20 = 100$

$P = 120N$

4. A Lorry of 3 tones pulls 2 trailers each of mass 2 tones along a horizontal road, if the lorry is accelerating at $0.8ms^{-2}$, calculate
   a) Net force acting on the whole combination
   b) The tension in the coupling between the lorry and 1st trailer.
   c) The tension in the coupling between the 1st and 2nd trailer.

Solution



For the lorry: $F_T - T_1 = 3000x0.8 = 2400.....(1)$

For 1st trailer: $T_1 - T_2 = 2000x0.8 = 1600.....(2)$

For 2ND trailer: $T_2 = 2000x0.8 = 1600N$

Put into [2]: $T_1 - T_2 = 1600$
$T_1 - 1600 = 1600$
$T_1 = 3200N$

Put into [1] $F - T_1 = 2400$
$F - 3200 = 2400$
$F = 5600N$

Exercise: 10

1. A large card board box of mass 0.75kg is pushed across a horizontal floor by a force of 4.5N. The motion of the box is opposed by a frictional force of 1.5N between the box and the floor, and an air resistance force given by kv^2 where $k = 6.0x10^{-2}kgm^{-1}$ and v is the speed of the box in m/s. calculate;
   (a) The acceleration of the box
   (b) Its speed **An(4.0m/s², 7.1m/s)**

2. A box of 50kg is pulled up from a ship with a[n] acceleration of $1ms^{-2}$ by a vertical rope attached to it.
   i. Find the tension on the rope.
   ii. What is the tension in the rope when th[e] moves up with a uniform velocity of 1m[/s]
   An [540N,490N]

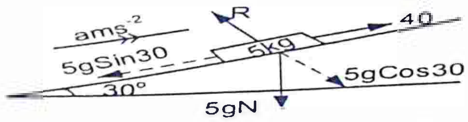
3. A lift moves up and down with an accelerat[ion] of $2ms^{-2}$. In each case, calculate the reaction [of] the floor on a man of mass 50kg standing i[n]

Motion on inclined planes

Examples

1. A body of mass 5kg is pulled up a smooth plane inclined at 30° to the horizontal by a force of 40N acting parallel to the plane. Find 4.
   a) Acceleration of the body
   b) Force exerted on the body by the plane

Solution



a) Resolving parallel to the plane: $F = ma$

$40 - 5gsin30 = ma$

$40 - 5x9.81sin30 = 5xa$

$a = 3.095ms^{-2}$

b) Force exerted on the body by the plane is the normal reaction 5.

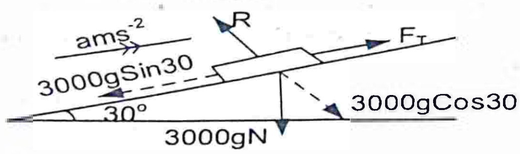
$R = 5gcos30 = 5x9.81cos30 = 42.4N$

2. A lorry of mass 3 tones travelling at 90km/h starts to climb an incline of 1 in 5. Assuming the tractive pull between its tyres and the road remains constant and that its velocity reduces to 54km/h in a distance of 500m. Find the tractive pull 6

Solution

$u = 90km/h = \frac{90x1000}{3600} = 25ms^{-1}$

$v = 54km/h = \frac{54x1000}{3600} = 15ms^{-1}$



Resolving along the plane

$F_T - 3000gsin\theta = 3000a$

$F_T - 3000x9.81x\frac{1}{5} = 3000a$

$F - 5886 = 3000a$..........................(i)

But $v^2 = u^2 + 2as$

$15^2 = 25^2 + 2ax500$

$a = -0.4ms^{-2}$

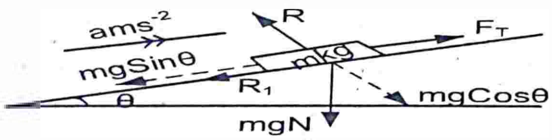
put into (i) $F - 5886 = 3000a$

$F = -3000x0.4 + 5886 = 4686N$

The tractive force is 4686N

3. A train travelling uniformly at 72km/h begins an ascent on 1 in 75. The tractive force which the engine exerts during the ascent is constant at 24.5kN, the resistance due to friction and air is also constant at 14.7kN, given the mass of the whole train is 225 tones. Find the distance a train moves up the plane before coming to rest.

Solution



1 in 75 means $sin\theta = \frac{1}{75}$ ∴ $\theta = 0.76°$

resistance force: $R_1 = 14.7kN$

tractive force: $F_T = 24.5kN$

$F_T - (mgsin\theta + R_1) = ma$

$24500 - (225000x9.81x\frac{1}{75} + 14700) = 22500$

$a = -0.087ms^{-2}$

its deceleration $= 0.087ms^{-2}$

$v^2 = u^2 + 2as$ [$v = 0m/s$ comes to rest]

$u = 72km/h = \frac{72x1000}{3600} = 20ms^{-1}$

$0^2 = 20^2 + 2(-0.087)s$

$-400 = -0.174s$

$S = 2298.85m$

Exercise: 11

1. The resistance to the motion of the train due to friction is equal to 1/160 of the weight of the train, if the train is travelling on a level road at $72kmh^{-1}$ and comes to the foot of an incline of 1 in 150 and steam is then turned off, how far will the train go up the incline before it comes to rest.
   An(1579.99m)
2. 12m length of the slope. If the truck starts from the bottom of the slope with a speed of 18km/h, how far up will it travel before coming to rest
   An(71.43m).
3. A car of 1 tonne accelerates from 36kmh to $72\ kmh^{-1}$ while moving $0.5kmh^{-1}$ up a road inclined at an angle of α to the horizontal, where $sin\alpha = \frac{1}{20}$. If the total resistive force to its motion is 0.3kN, find the driving force of the car engine
   An(1009N).

4. A railway truck of mass 6.0 tonnes moves with an acceleration of 0.050ms^{-2} down a track which is inclined to the horizontal at an angle α where $sin\alpha = \frac{1}{120}$. Find the resistance to motion **An(2.0x10^{2}N).**

5. A body of mass 5.0kg is pulled along a smooth horizontal ground by means of force of 40N acting at 60° above the horizontal. Find
   (a) Acceleration of the body
   (b) Force the body exerts on the ground
   An(4.0ms^{-2}, 15.4N).

6. A railway engine of mass 100 tones is attached to a line of truck of total mass 80 tones. Assuming there is no resistance to motion, find the tension in the coupling between the engine and the leading truck when the train
   (a) has an acceleration of 0.020ms^{-2}
   (b) Is moving at constant velocity **An(25.6kN).**

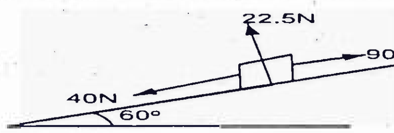
7. A5000kg engine pulls a train of 5 trucks, each of 2000kg along a horizontal track. If the engine exerts a force of $50,000N$ and the frictional resistance is $5000N$, calculate;
   (i) Net accelerating force
   (ii) Acceleration of the train
   (iii) Force of truck 1 on truck 2
   An(45,000N, 3.0ms^{-2}, 24,000N).

8. A body of mass 3.0kg slides down a plane which is inclined at 30° to the horizontal. Find the acceleration no of the body , if:
   (a) The plane is smooth
   (b) There is a frictional resistance of 9.0N
   An(5.0ms^{-2},2.0ms^{-2}).

9. A car of mass 1000kg tows a caravan of mass 600kg up a road which rises 1m vertically for every 20m of its length. There are constant frictional resistance of 200N and 100N to the motion of the car and to the motion of the caravan respectively. The combination has an acceleration of 1.2ms^{-2} with the engine exerting a constant driving force. Find
   (a) Driving force
   (b) Tension in the tow- bar **An(3.02kN, 1.12kN).**

10. A 25kg block rests at the top of a smooth plane whose length is 2.0m and whose height at elevated end is 0.5m. how long will it take for the block to slide to the bottom of plane when released **An(1.258s)**

11. Three forces act on a block as shown, the block is placed on a smooth plane inclined at 60°



calculate;
a) Acceleration of the block up the plane
b) Gain in kinetic energy in 5s after moving from rest **An(1.5ms^{-2},140.625J)**

4.1.1: Motion of connected particles

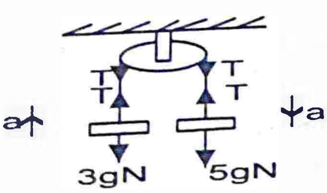
When two particles are connected by a light inextensible string passing over a smooth pulley and allowed to move freely, then as long as the string is tight, the following must be observed.

- Acceleration of one body in general direction of motion is equal to the acceleration of the other
- The tension T in the string is constant.

Examples

1. Two particles of masses 5kg and 3kg are connected by a light inelastic string passing over a smooth fixed pulley. Find;
   (i) Acceleration of the particles
   (ii) The tension in the string
   (iii) The force on the pulley

Solution



Using $F = ma$

For 5kg mass: $5g - T = 5a$..............(i)

For 3kg mass: $T - 3g = 3a$ ....(ii)

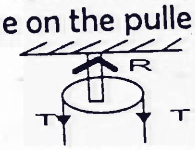
$2g = 8a$

$a = \frac{2x9.81}{8} = 2.45ms^{-2}$

ii) $T - 3g = 3a$

$T = 3x2.45 + 3x9.81 = 36.78N$

iii) Force on the pulley

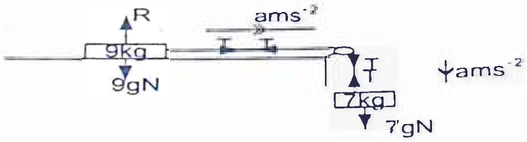


$R = 2T = 2x36.78 = 73.56N$

Force on the pulley is 73.56N

2. A mass of 9kg resting on a smooth horizontal table is connected by a light string passing over a smooth pulley at the edge of the table to the pulley is a 7kg mass hanging freely; find
 (i) Common acceleration
 (ii) The tension in the string
 (iii) The force on the pulley in the system if its allowed to move freely.

Solution



Using $F = ma$

For 7kg mass: $7g - T = 7a$.............(i)

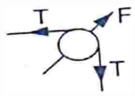
For 9kg mass: $T = 9a$...........................(ii)

Put (ii) into (i): $7g - 9a = 7a$

$a = \frac{7g}{16} = \frac{7x9.81}{16} = 4.292ms^{-2}$

(ii) Tension: $T = 9a = 9x4.292 = 38.63N$

(iii) The force on the pulley



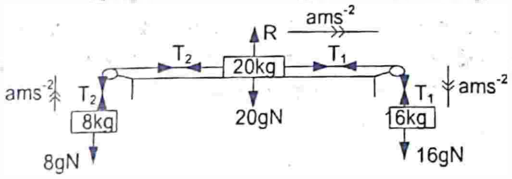
$F = \sqrt{T^2 + T^2} = T\sqrt{2} = 38.63\sqrt{2}$

Force on the pulley $= 54.63N$

3.

The figure shows a block of mass 20 kg resting on a smooth horizontal table. Its connected by strings which pass over pulleys at the edges of the table to two loads of masses 8kg and 16kg which hang vertically. Calculate;
 (i) Acceleration of 16kg mass
 (ii) Tension in each string
 (iii) Reaction on each pulley

Solution



Using $F = ma$

For 16kg mass $16g - T_1 = 16a$..........[1]

For 20kg mass $T_1 - T_2 = 20a$............[2]

For 8kg mass $T_2 - 8g = 8a$...............[3]

Adding 1 and 2: $16g - T_2 = 36a$.............[x]

And (3) and (x): $8g = 44a$

$a = \frac{8x9.81}{44} = 1784ms^{-2}$

ii) Tension in each string

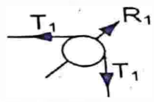
$16g - T_1 = 16a$

$T_1 = 16x9.81 - 16x1.784 = 128.416N$

$T_2 - 8g = 8a$

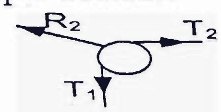
$T_2 = 8x1.784 + 8x9.81 = 92.752N$

iii) Reaction on each pulley



$R_1 = \sqrt{T_1^2 + T_1^2} = T_1\sqrt{2} = 128.416x\sqrt{2}$

$R_1 = 181.61N$



$R_2 = T_2\sqrt{2} = 92.752\sqrt{2} = 131.171N$

Exercise12

1. Two particles of masses 7kg and 3kg are connected by a light inelastic string passing over a smooth fixed pulley. Find;
 (i) Acceleration of the particles
 (ii) The tension in the string
 (iii) The force on the pulley **An(3.92ms⁻², 41.16N, 82.32N)**

2. Two particles of masses 6kg and 2kg are connected by a light inextensible string passing over a smooth fixed pulley. With the masses hanging vertically, system is released from rest. Find;
 (i) Acceleration of the particles
 (ii) The tension in the string
 (iii) Distance moved by the 6kg mass in the first 2 seconds of motion
 An(4.9ms⁻², 3N, 9.8m)

3. A man of mass 70kg and a bucket of bricks of mass 100kg are tied to the opposite ends of a

which passes over a frictionless pulley so that they hang vertically downwards

(a) what is the tension in the section of the section of rope supporting the man

(b) What is the acceleration of the bucket

An(807.06N, 1.73ms^{-2})

4. Two particles of masses 20g and 30g are connected to a fine string passing over a smooth pulley, when released freely find;

(i) Common acceleration

(ii) The tension in the string

(iii) The force on the pulley

An [1.962ms^{-2} ,0.235N ,0.471N]

5. A mass of 5kg is placed on a smooth horizontal table and connected by a light string to a 3kg mass passing over a smooth pulley at the edge of the table and hanging freely. If the system is allowed to move, calculate;

a) The common acceleration of the masses

b) The tension in the string

c) The force acting on the pulley

An[3.68m/s^2, 18.4N, 26N]

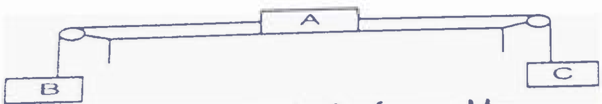
6. Two objects of mass 3kg and 5kg are attached to the ends of a cord which passes over a fixed frictionless pulley placed at 4.5m above the floor. The objects are held at rest with 3 kg mass touching the floor and the 5kg mass at 4m above the ground and then released, what is

(i) The acceleration of the system. **An(2.45ms^{-2}).**

(ii) The tension of the cord **An(36.75N).**

(iii) Time will elapse before the 5kg object hits the floor **An(1.81s).**

7.



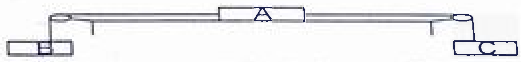
The diagram shows a particle A of mass M = 2kg resting on a horizontal table. It is attached to particles B of m =5kg and C of m= 3kg by light inextensible strings hanging over light smooth pulleys. If the system is allowed to move from rest, find the common acceleration of the particle and the tension in each string given that the surface of the table is rough and the coefficient of friction between the particle and the surface of the table is ½

An[0.98ms^2, 32.37N, 44.15N]

8.

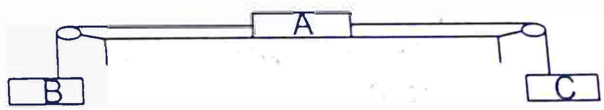
The diagram shows a particle A of mass $2kg$ resting on a rough horizontal table of coefficient of friction 0.5. It is attached to particles B of mass $5kg$ and C of mass $3kg$ by light inextensible strings hanging over light smooth pulleys. If the system is allowed to move from rest, find the common acceleration of the particle and the tension in each string. **An[0.98ms^2, 32.37N, 44.15N]**

9.



The diagram shows a particle A of mass $5kg$ resting on a rough horizontal table. It is attached to particles B of mass $3kg$ and C of mass $2kg$ by light inextensible strings hanging over light smooth pulleys. If the system is released from rest, body B descends with an acceleration of 0.28ms^{-2}, find the c coefficient of friction between the body A and the surface of the table **An[$\frac{1}{7}$]**

10.



The diagram shows a particle A of mass $10kg$ resting on a smooth horizontal table. It is attached to particles B of mass $4kg$ and C of mass $7kg$ by light inextensible strings hanging over light smooth pulleys. If the system is allowed to move from rest, find the common acceleration of the particle and the tension in each string. **An [1.4ms^{-2},44.8N, 58.8N]**

4.1.2: LINEAR MOMENTUM AND IMPULSE

Momentum is the product of mass and velocity of the body moving in a straight line

Momentum (p) =mass x velocity

$$\vec{p} = m\vec{v}$$

Momentum is a vector quantity

Definition: Linear momentum (p) is the product of the mass and the velocity of the body moving in

IMPULSE

This is the product of the force and time for which the force acts on a body

i.e. Impulse (I) = Force(F) x time (t) $\vec{I} = \vec{F}t$

The unit of impulse is Ns.

An impulse produces a change in momentum of a body. If a body of mass(m) has it velocity changed from u to v by a force F acting on it in time t, then from Newton's 2[nd] law.

$Ft = mv - mu$

$I = Ft$

$I = mv - mu$

Impulse = change in momentum

Examples

1. A body of mass 5kg is initially moving with a constant velocity of $2ms^{-1}$, when it experiences a force of 10N is 2s, find
   (i) The impulse given to the body by the force
   (ii) The velocity of the body when the force stops acting

Solution

$I = ft = 10x2 = 20Ns$

$I = mv - mu$

$20 = 5v - 5x2$

$v = 6m/s$

2. A girl of mass 50kg jumps onto the ground from a height of 2m. Calculate the force which acts on her when she lands
   (i) As she bends her knees and stops within 0.2 s
   (ii) As she keeps her legs straight and stops in 0.05s

Solution

i) $v^2 = u^2 + 2gs$

$v^2 = 0^2 + 2x9.81x2$

$v = \sqrt{39.24} = 6.03ms^{-1}$

Using $F = \frac{mv - mu}{t}$

$F = \frac{50(6.03-0)}{0.2} = 1507.5N$

ii) $F = \frac{mv - mu}{t}$

$F = \frac{50(6.03-0)}{0.05} = 6030N$

4.1.3: WHY LONG JUMPER BEND KNEES

By bending the knees, the time taken to come to rest is increased, which reduces the rate of change of momentum, therefore the force on the jumpers legs is reduced thus less pain on the legs.

Questions

1. Explain why, when catching a fast moving ball, the hands are drawn backwards while ball is being brought to rest.
2. Explain why a long jumper must land on sand
3. Why is it much more painful to be hit by a hailstone of mass 0.005kg falling at 5m/s which bounces off your head than by a raindrop of the same mass and falling at the same velocity but which breaks up on hitting you and does not bounce?(numerical answered is required)

4.1.4: LAW OF CONSERVATION OF LINEAR MOMENTUM

It states that for a system of colliding bodies, their total linear momentum remains constant in a given direction provided no external forces acts on them.

Suppose a body A of mass m, and velocity U_1, collides with another body B of mass m_2 and velocity U_2 moving in the same direction

By principle of conservation of momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$m_1u_1 + m_2u_2$: Total momentum before collision

$m_1v_1 + m_2v_2$: Total momentum after collision

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… of the law of conservation of momentum using Newton's law

Let two bodies A and B with masses m_1 and m_2 moving with initial velocities u_1 and u_2 and let their velocities after collision be v_1 and v_2 respectively for time t with ($v_1<v_2$)

By Newton's 2^{nd} law:

Force on m_1: $F_1 = \frac{m_1(v_1-u_1)}{t}$

Force on m_2: $F_2 = = \frac{m_2(v_2-u_2)}{t}$

By Newton's 3rd law: $F_1 = -F_2$

$$\frac{m_1(v_1-u_1)}{t} = -\frac{m_2(v_2-u_2)}{t}$$

$$m_1v_1 - m_1u_1 = -m_2v_2 + m_2u_2$$

$$\therefore\ m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

Hence $m_1u_1 + m_2u_2 =$ **constant**

4.1.6: COLLISIONS

In an isolated system, momentum is always conserved but this is not always true of the kinetic energy of the colliding bodies.

In many collisions, some of the kinetic energy is converted into other forms of energy such as heat, light and sound.

Types of collisions

1. **Elastic collisions**

   It is also perfectly elastic collision. This is a type of collision in which all kinetic energy is conserved. *Eg* collision between molecules, electrons.

2. **Inelastic collision**

   This is a type of collision in which the kinetic energy is not conserved.

3. **Completely inelastic collision**

   This is a type of collision in which the bodies stick together after impact and move with a common velocity. *Eg* a bullet embedded in a target

4. **Explosive collision (super elastic)**

   This is one where there is an increase in K.E.

Summary

Elastic collision

- Linear momentum is conserved
- Kinetic energy is conserved
- Bodies separate after collision
- Coefficient of restitution (elasticity)=1 (e=1)

Inelastic collision

- Linear momentum is conserved
- K.e is not conserved
- Bodies separate after collision
- Coefficient of restitution is less than 1 (e<1)

Perfectly inelastic

- Linear momentum is conserved
- K.e is not conserved
- Bodies stick together and move with a common velocity
- e=0

4.1.7: Mathematic treatment of elastic collision

Consider an object of mass m, moving to the right with velocity u_1. If the object makes a head-… elastic collision with another body of mass m_2 moving with a velocity u_2 in the same directi… Let v_1 and v_2 be the velocities of the two bodies after collision.

By conservation of momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$m_1(u_1 - v_1) = m_2(v_2 - u_2) \text{----------------[1]}$$

For elastic collision k.e is conserved

$$½\, m_1u_1^2 + ½\, m_2u_2^2 = ½\, m_1{v_1}^2 + ½\, m_2{v_2}^2$$

$$m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2) \text{ --------}$$

from equation 1 and 2 then

$$\frac{m_1(u_1 - v_1)}{m_1(u_1^2 - v_1^2)} = \frac{m_2(v_2 - u_2)}{m_2(v_2^2 - u_2^2)}$$

$$\frac{(u_1 - v_1)}{(u_1 + v_1)(u_1 - v_1)} = \frac{(v_2 - u_2)}{(v_2 + u_2)(v_2 - \ldots}$$

$$\frac{1}{(u_1 + v_1)} = \frac{1}{(v_2 + u_2)}$$

$$u_1 + v_1 = v_2 + u_2$$

$$v_2 - v_1 = -(u_2 - u_1)$$

Examples

1. A particle P of mass m_1, travelling with a speed u_1 makes a head-on collision with a stationary particle Q of mass m_2. If the collision is elastic and the speeds of P and Q after impact are v_1 and v_2 respectively. Show that for $\beta = \frac{m_1}{m_2}$

(i) $\frac{u_1}{v_1} = \frac{\beta+1}{\beta-1}$

(ii) $\frac{v_2}{v_1} = \frac{2\beta}{\beta-1}$

Solution

By law of conservation of momentum

$m_1u_1 = m_1v_1 + m_2v_2$ ------------[x]

$(u_1 - v_1) = \frac{m_2}{m_1}v_2$

Therefore $u_1 - v_1 = \frac{v_2}{\beta}$

$\beta(u_1 - v_1) = v_2$ ---------------------------- [1]

for elastic collision k.e is conserved

$\frac{1}{2} m_1u_1^2 = \frac{1}{2}m_1{v_1}^2 + \frac{1}{2} m_2{v_2}^2$

$m_1(u_1^2 - v_1^2) = m_2({v_2}^2)$

$\frac{m_1}{m_2}({u_1}^2 - {v_1}^2) = {v_2}^2$

$\beta(u_1^2 - v_1^2) = v_2^2$ ----------------------------[2]

equating [1] and [2]

$\beta(u_1^2 - v_1^2) = [\beta(u_1 - v_1)]^2$

$\beta(u_1^2 - v_1^2) = \beta^2(u_1 - v_1)(u_1 - v_1)$

$(u_1 - v_1)(u_1 + v_1) = \beta(u_1 - v_1)(u_1 - v_1)$

$(u_1 + v_1) = \beta(u_1 - v_1)$

$v_1 + \beta v_1 = \beta u_1 - u_1$

$v_1(1 + \beta) = u_1(\beta - 1)$

$\frac{u_1}{v_1} = \frac{\beta+1}{\beta-1}$ ----------------------------[xx]

ii) From $\frac{u_1}{v_1} = \frac{\beta+1}{\beta-1}$

from equation [1]: $v_2 = \beta(u_1 - v_1)$

$v_2 = \beta u_1 - \beta v_1$

$u_1 = \frac{v_2 + \beta v_1}{\beta}$ put into (xx)

$$\frac{\left(\frac{v_2 + \beta v_1}{\beta}\right)}{v_1} = \frac{(1+\beta)}{(\beta - 1)}$$

$(v_2 + \beta v_1)(\beta - 1) = (1 + \beta)\beta v_1$

$\beta v_2 + \beta^2 v_1 - v_2 - \beta v_1 = \beta v_1 + \beta^2 v_1$

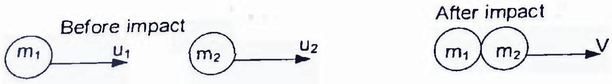
$\beta v_2 - v_2 = 2\beta v_1$

$v_2(\beta - 1) = 2\beta v_1$

$\frac{v_2}{v_1} = \frac{2\beta}{\beta - 1}$

4.1.8: Mathematical treatment of perfectly inelastic collision

Suppose a body of mass m, moving with velocity u_1 to the right makes a perfectly inelastic collision with a body of mass m_2 moving with velocity u_2 in the same direction



By law of conservation

$$m_1u_1 + m_2u_2 = (m_1 + m_2)v$$

$$v = \frac{m_1u_1 + m_2u_2}{m_1 + m_2}$$

$k.e_{initial} = \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2$

$k.e_{final} = \frac{1}{2}(m_1 + m_2)v^2$

Loss in $k.e = k.e_{initial} - k.e_{final}$

$$= \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 - \frac{1}{2}(m_1 + m_2)v^2$$

Numerical examples

1. Ball P, Q and R of masses m_1, m_2 and m_3 lie on a smooth horizontal surface in a straight line. The balls are initially at rest. Ball P is projected with a velocity u_1 towards Q and makes an elastic collision with Q. if Q makes a perfectly in elastic collision with R, show that R moves with a velocity.

$$v_2 = \frac{2m_1m_2u_1}{(m_1 + m_2)(m_2 + m_3)}$$

Solution

Elastic collision of P and Q:

Conservation of momentum:

$m_1u_1 = m_1v_P + m_2v_Q$

$v_P = u_1 - \frac{m_2v_Q}{m_1}$.......(1)

Conservation of kinetic energy:

$\frac{1}{2}m_1u_1^2 = \frac{1}{2}m_1v_P^2 + \frac{1}{2}m_2v_Q^2$.....(2)

Putting [1] into [2

$$m_1u_1^2 = m_1\left(u_1 - \frac{m_2v_Q}{m_1}\right)^2 + m_2v_Q^2$$

$v_Q = \frac{2m_1u_1}{m_1+m_2}$......(3)

In elastic collision of Q and R:

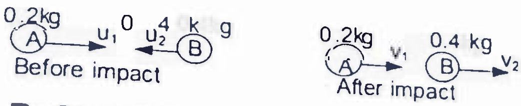
$m_2 v_Q + m_3 0 = (m_2 + m_3)v_2$

$m_2 \frac{2m_1u_1}{m_1+m_2} = (m_2 + m_3)v_2$

$v_2 = \frac{2m_1m_2u_1}{(m_1+m_2)(m_2+m_3)}$

2. A 0.2kg block moves to the right at a speed of 1ms⁻¹ and meets a 0.4kg block moving to the left with a speed of 0.8ms⁻¹. Find the find velocity of each block if the collision is elastic.

Solution



By law of conservation

$M_1U_1 + M_2U_2 = M_1V_1 + M_2V_2$

$(0.2x1) + (0.4x - 0.8) = 0.2v_1 + 0.4v_2$

$0.2 - 0.32 = 0.2v_1 + 0.4v_2$

$v_1 + 2v_2 = -0.6$---------------------- [1]

for elastic collision K.E is conserved

$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$

$0.2x1^2 + 0.4x(-0.8)^2 = 0.2v_1^2 + 0.4v_2^2$

$0.2 + 0.256 = 0.2v_1^2 + 0.4v_2^2$

$v_1^2 + 2v_2^2 = 2.28$----------------------[2]

But from [1] $v_1 = -0.6 - 2v_2$ put into (2)

$v_1^2 + 2v_2^2 = 2.28$

$2v_2^2 + (-0.6 - 2v_2)^2 = 2.28$

$6v_2^2 + 2.4v_2 - 1.92 = 0$

$v_2 = 0.4m/s, v_2 = -0.8m/s$

$v_2 = 0.4m/s$ is correct since m_2 is in front it supposed to move faster

Therefore from (1)

$v_1 + 2v_2 = -0.6$

$v_1 + 2(0.4) = -0.6$

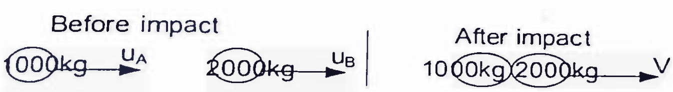
$v_1 = -1.4m/s$

3. A truck of mass 1 tonne travelling at 4m/s collides with a truck of mass 2 tonnes moving at 3m/s in the same direction. If the collision is perfectly inelastic, calculate;

(i) Common velocity

(ii) Kinetic energy converted to other forms during collision

Solution



By law of conservation of momentum

$M_AU_A + M_BU_B = (M_A + M_B)V$

$(1000x4) + (2000x3) = (1000 + 2000)v$

$V = 3.3333ms^{-1}$

ii) Initial K.e $= \frac{1}{2}M_AU_A^2 + \frac{1}{2}M_BU_B^2$

$= ½x1000x4^2 + ½x2000x3^2 = 17000J$

Final k.e$_f$ $= \frac{1}{2}(M_A + M_B)V^2$

$= ½(1000 + 2000)(3.3333)^2$

$= 16666.67J$

Kinetic energy converted $= k.e_{initial} - k.e_{final}$

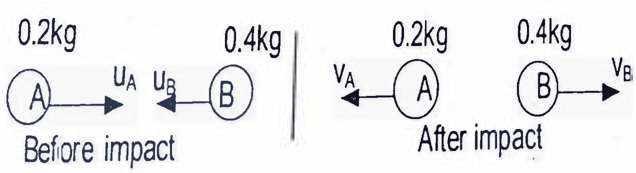
$= 17000 - 16666.67 = 333.33Joules$

4. Two particles of masses 0.2kg and 0.4kg are approaching each other with velocities 4ms⁻¹ and 3ms⁻¹ respectively. On collision, the first particle reverses, its direction and moves with a velocity of 2.5ms⁻¹ find the;

(i) velocity of the second particle after collision

(ii) percentage loss in kinetic energy

Solution



By law of conservation of momentum

$M_AU_A + M_BU_B = M_AV_A + M_BV_B$

$0.2x4 + 0.4x - 3 = 0.2x2.5 + 0.4V_B$

$V_B = 0.25m/s$

ii) Initial k.e$_i$ $= \frac{1}{2}M_AU_A^2 + \frac{1}{2}M_BU_B^2$

$= ½(0.2x4^2 + 0.4x[-3]^2) = 3.4J$

Final K.e$_f$ $= \frac{1}{2}M_AV_A^2 + \frac{1}{2}M_BV_B^2$

$= \frac{1}{2}x\ 0.2x2.5^2 + \frac{1}{2}x0.4x0.25^2 = 0.6475J$

Loss in kinetic energy = k.e$_i$ − k.e$_f$

$= 3.4 - 0.6375 = 2.7625J$

% loss in k.e. $= \frac{loss\ of\ k.e}{k.e_i} x100\%$

$= \frac{2.7625}{3.4} x100\% = 81.25\%$

5. A bullet of mass $1.5 \times 10^{-2} kg$ is fired from a rifle of mass $2.7 \times 10^{2} kg$ with a muzzle velocity of 100km/h. Find the recoil velocity of the rifle.

Solution

$$V_b = \frac{100 \times 1000}{60 \times 60} = 27.78 m/s$$

$$M_g V_g = M_b V_b$$

$$3V_g = 1.5 \times 10^{-2} \times 27.78$$

$$V_g = 0.14 m/s$$

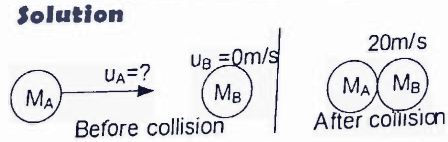
9.

6. A bullet of mass 20g is fired into a block of wood of mass 400g lying on a smooth horizontal surface. If the bullet and the wood move together with the speed of 20m/s. Calculate

(i) The speed with which the bullet hits the wood

(ii) The kinetic energy lost

Solution



$$M_A U_A + M_B U_B = (M_A + M_B)V$$

$$(0.02 x U_A) + (0.4 x 0) = (0.02 + 0.4) x 20$$

$$u_A = 420 m/s$$

The original velocity of the bullet was *420m/s*

Initial k.e $= \frac{1}{2} M_A U_A^2 + \frac{1}{2} M_B U_B^2$

$= \frac{1}{2} x\, 0.02 x\, 420^2 + \frac{1}{2} x 0.4 x 0^2 = 1764J$

Final k.e $= \frac{1}{2}(M_A + M_B)V^2$

$= \frac{1}{2} x (0.02 + 0.04) x (20)^2 = 84J$

Loss in kinetic energy $=$ k.e$_i$ $-$ k.e$_f$

$= 1764 - 84 = 1680J$

7. A particle **P** of mass m_1 moving at a speed u_1 collides head on with a stationery particle Q of mass m_2. the collision is perfectly elastic and the speeds of P and Q after impact are v_1 and v_2 respectively. Given that $\alpha = \frac{m_2}{m_1}$

(i) Determine the value of α if $u_1 = 20v_2$

(ii) Show that the fraction of energy lost by P is $\frac{4\alpha}{(1+\alpha)^2}$

Solution

(i) $m_1 u_1 = m_1 v_1 + m_2 v_2$

$m_1(u_1 - v_1) = m_2 v_2$

$(u_1 - v_1) = \alpha v_2$ ....(1)

$v_1 = u_1 - \alpha v_2$ ....(2)

$\frac{1}{2} m_1 u_1^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$

$m_1(u_1^2 - v_1^2) = m_2(v_2^2)$

$(u_1^2 - v_1^2) = \alpha\, v_2^2$ ---------------- [3]

equating [3] ÷ [1]: $\frac{\alpha(u_1^2 - v_1^2)}{\alpha(u_1 - v_1)} = \frac{\alpha v_2^2}{\alpha v_2}$

$$\frac{(u_1 - v_1)(u_1 + v_1)}{(u_1 - v_1)} = \frac{v_2^2}{v_2}$$

$(u_1 + v_1) = v_2$ ....(4)

Put (2) into (4): $(u_1 + u_1 - \alpha v_2) = v_2$

$2u_1 = (1 + \alpha) v_2$ ....(5)

but $u_1 = 20v_2$

$40v_2 = (1 + \alpha)\, v_2$

$\alpha = 39$

(iii) k.e of p before collision $= \frac{1}{2} m_1 u_1^2$

k.e of p after collision $= \frac{1}{2} m_1 v_1^2$

energy lost $= \frac{1}{2} m_1 u_1^2 - \frac{1}{2} m_1 v_1^2$

fraction of energy lost $= \frac{\frac{1}{2} m_1 u_1^2 - \frac{1}{2} m_1 v_1^2}{\frac{1}{2} m_1 u_1^2}$

fraction of energy lost $= \frac{(u_1^2 - v_1^2)}{u_1^2} = \frac{(u_1 - v_1)(u_1 + v_1)}{u_1^2}$

from (i) above $(u_1 + v_1) = v_2, (u_1 - v_1) = \alpha v_2$

$u_1 = \frac{(1+\alpha)}{2} v_2$

fraction of energy lost $= \frac{(\alpha v_2)(v_2)}{\left[\frac{(1+\alpha)}{2} v_2\right]^2} = \frac{4\alpha}{(1+\alpha)^2}$

8. A body explodes and produces two fragments of masses m and M. If the velocities of the fragments are u and v respectively, show that the ratio of kinetic energies of the fragments is

$$\frac{E_1}{E_2} = \frac{M}{m}$$

Where E_1 is the kinectic energy of m and E_2 is the kinectic energy of M

Solution

$$E_1 = \frac{1}{2} m u^2 \quad and \quad E_2 = \frac{1}{2} M v^2$$

By law of conservation of linear momentum:

$$mu = -Mv$$

$$\therefore v = \frac{-mu}{M}$$

$$E_2 = \frac{1}{2} M \left(\frac{-mu}{M}\right)^2 = \frac{1}{2} \frac{m^2 u^2}{M}$$

$$\frac{E_1}{E_2} = \frac{\left(\frac{1}{2} mu^2\right)}{\left(\frac{1}{2} m^2 u^2 \ldots\right)} = \frac{M}{\ldots}$$

9. An object X of mass 2kg, moving with a velocity 10ms^{-1} collides with a stationary object Y of equal mass. After collision X moves with speed U at an angle of 30° to its initial direction while Y moves with a speed of Y at an angle of 90° to the new direction.
   (i) Calculate the speeds U and V (05marks)
   (ii) Determine whether the collision is elastic or not. (03marks)

Solution

$(\rightarrow): 2x10 = 2u\cos 30 + 2v\cos 60$

$$20 = 2u\frac{\sqrt{3}}{2} + 2v\frac{1}{2}$$

$v = 20 - u\sqrt{3}$ .... [1]

$(\uparrow): 0 = 2u\sin 30 - 2v\sin 60$

$2u\sin 30 = 2v\sin 60$

$$\frac{u}{2} = v\frac{\sqrt{3}}{2}$$

$u = v\sqrt{3}$.............[2]

Put into [1]: $v = 20 - \sqrt{3}\, v\sqrt{3}$

$4v = 20$

$v = 5ms^{-1}$

$u = v\sqrt{3} = 5\sqrt{3} = 8.66ms^{-1}$

i. Total K.E before collision

$K.e = \frac{1}{2}x2x10^2 = 100J$

Total K.e after collision

$= {}^1/_2 x2x(5)^2 + 1/2x2x(5\sqrt{3})^2 = 100J$

Since kinetic energy is conserved then the collision is elastic

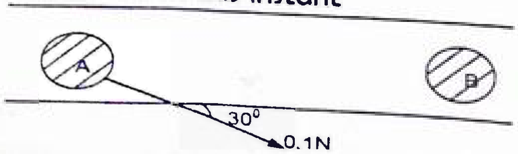
Exercise:13

1. A 4kg ball moving at 8m/s collides with a stationery ball of mass 12kg, and they stick together. Calculate the final velocity and the kinetic energy lost in impact **An [2m/s, 96J]**
2. A body of mass 6kg moving at 8ms^{-1} collides with a stationary body of mass 10kg and sticks to it. Find the sped of the composite body immediately after impact **An(3m/s)**
3. A bullet of mass 6g is fired from a gun of mass 0.50kg. if the muzzle velocity of the bullet is 300ms^{-1}, calculate the recoil velocity of the gun **An(3.6m/s)**
4. A body A of mass 4kg moves with a velocity of 2ms^{-1} and collides head on with another body, B of mass 3kg moving in the opposite direction at 5ms^{-1}. After the collision the bodies move off together with v. Calculate v **An(-1m/s)**
5. Amass A of 6kg moving a velocity of 5m/s collides with a mass B of mass 8kg moving in the opposite direction at 3m/s.
   (a) calculate the final velocity if the masses stick together on impact
   (b) If the masses do not stick together but mass A continues along the same direction with a velocity of 0.5m/s after impact. Calculate the velocity of B. **An (0.43m/s, 0.38m/s)**
6. A sphere of mass 3kg moving with velocity 4m/s collides head–on with a stationary sphere of mass 2kg and imparts to it a velocity of 4.5m/s. calculate the;
   (i) velocity of the 3kg sphere after the collision.
   (ii) amount of energy lost by the moving bodies in the collision **An (1m/s, 2.25J)**
7. A 2kg object moving with a velocity of 8m/s collides with a 3kg object moving with a velocity 6ms^{-1} along the same direction. If the collision is completely inelastic, calculate the decrease in kinetic energy collision. **An [2.4J]**
8. Two bodies A and B of mass 2kg and 4kg moving with velocities of 8m/s and 5m/s respectively collide and move on in the same direction. Object A's new velocity is 6m/s.
   (i) Find the velocity of B after collision
   (ii) Calculate the percentage loss in kinetic energy. **An(6m/s,5.26%)**
9. A railway truck of mass 4x10^4 kg moving at a velocity of 3m/s collides with another truck of mass 2x10^{4}kg which is at rest. The coupling join and the trucks move off together
   (i) What fraction of the first trucks initial kinetic energy remains as kinetic energy of two trucks after collision **An [$^2/_3$]**
   (ii) Is energy conserved in a collision such as this, explain your answer
10. A particle of mass 2kg moving with speed 10ms collides with a stationary particle of mass 7kg. Immediately after impact the particles move w the same speeds but in opposite directions. Fin loss in kinetic energy during collision. **An(28**

11. A bullet of mass 2.0x10⁻³kg is fired horizontally into a free- standing block of wood of mass 4.98x10⁻¹kg, which it knocks forward with an initial speed of 1.2m/s
    (a) Estimate the speed of the bullet
    (b) How much kinetic energy is lost in the impact **An(300m/s, 89.64J)**
    (c) What becomes of the lost kinetic energy
12. A 2kg object moving with a velocity of 6ms⁻¹ collides with a stationary object of mass 1kg. If the collision is perfectly elastic, calculate the velocity of each object after collision. **An[2m/s, 8ms⁻¹]**
13. A body of mass **m** makes a head on , perfectly elastic collision with a body of mass **M** initially at rest. Showthat $\frac{\Delta E}{E_0} = \frac{4\left(\frac{M}{m}\right)}{\left(1+\frac{M}{m}\right)^2}$ where $E_0$is original kinetic energy of the mass **m** and ΔE the energy it loses in the collision
14. A metal sphere of mass m_1, moving at velocity u_1 collides with another sphere of mass $m_2$moving at velocity $u_2$in the same direction. After collision the spheres stick together and move off as one body. Show that the loss in kinetic energy E during collision is given by

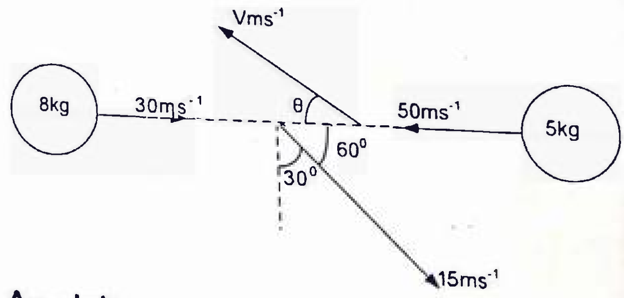
$$E = \frac{\beta(u_1 - u_2)^2}{2\,(m_1 + m_2)} \quad where\,\beta = m_1 m_2$$

15. A stationary radioactive nucleus disintegrates into an α −particle of relative atomic mass 4, and a residual nucleus of relative atomic mass 144. If the kinetic energy of the α −particle is 3.24x10⁻¹³J, what is the kinetic energy of the residual nucleus **An(9x10⁻¹⁵J)**
16. On a linear air-track the gliders float on a cushion of air and move with negligible friction. One such glider of mass 0.50kg is at rest on a level track. A student fires an air rifle pellet of mass 1.5x10⁻³kg at the glider along the line of the track. The pellet embeds it's in the glider which recoil with a velocity of 0.33m/s. calculate the velocity ta which the pellet struck **An(110m/s)**
17. The diagram below shows a body A of mass 2kg resting in a frictionless horizontal gully in which it is constrained to move. It is acted upon by a force shown below for 5s after which time it strikes and sticks to the body B of mass 3kg, the force being removed at this instant



what will the speedof the combined masses be.
An(0.08m/s)

18. A kitten of mass0.6kg leaps at 30° to the horizontal out of a toy truck of mass 1.2kg causing it to move over a horizontal ground at 4.0ms⁻¹. At whatsped did the kitten leap
An(9.2m/s)
19. A flat truck of mass 400kg is moving freely along a horizontal track at 3.0ms⁻¹. A man moving at right angles to the track jumps on to the truck causing its speed to decrease by 0.50ms⁻¹. What the mass of the man. **An(80kg)**
20. A proton of mass $16x10^{-27}kg$ travelling with a velocity of $3x10^7ms^{-1}$ collides with the nucleus of a stationary oxygen atom of mass $2.56x10^{-26}kg$ and rebounds in a direction at 90° to its original path. Calculate the velocity and direction of the oxygen nucleus, assuming the collision is perfectly elastic.
An$(2.65x10^6m/s,\ 45^0$ to the original direction of the proton)
21. A ball A of mas 10kg moving with a speed of $8ms^{-1}$ collides with another ball B of mass 20k initially at rest. After collision, A and B move in direction making angles of 30° and 45° respectively with the initial direction of mo of A. calculate the speed of A and B after the collision **An**$(5.85ms^{-1},\ 2.07ms^{-1})$
22. Two balls collide and bounce off each other c shown below. Determine the final velocity v of 5kg mas if 8kg mass has a speed of $15ms^{-1}$ after collision.



23. An alpha particle of mass 4 units is incident with a velocity u on a stationary helium nucleus of equal mass. After collision, an alp particle moves with a velocity $\frac{u}{2}$ at an angle 60° to its initial direction while the helium nucleus moves at angle θ to the initial direct of the alpha particle. Calculate the velocity the helium nucleus after collision and the va ofθ. An$(\frac{u\sqrt{3}}{2}ms^{-1}, \theta = 30°)$

Application of law of conservation of momentum

Consider a horse pipe of cross-sectional area A giving a water jet of velocity v, if the water hits the wall and comes to rest then;

$$mass\,of\ water\ striking\ the\ wall\ per\ second = \rho v A$$

Where ρ is density of water

Force due to water $= mass\ per\ second\ x velocity\ change$

$$\boxed{Force\ = \rho A v^2}$$

Examples

1. Water leaves horse pipe at a rate of $5.0kgs^{-1}$ with a speed of $20ms^{-1}$ and is directed horizontally on a wall which stops it. Calculate the force exerted by the water on the wall.

   Solution

   Force due to water= $mass\ per\ second\ x velocity\ change = 5\,x(20-0) = 100N$

2. A horse pipe has a hole of cross-sectional area $50cm^2$ and ejects water horizontally at a speed of $0.3ms^{-1}$. If the water is incident on a vertical wall and its horizontal velocity becomes zero. Find the force the water exerts on the wall.

   Solution

   Force due to water= $\rho A v^2 = 0.3x50x10^{-4}x1000x0.3 = 0.45N$

3. Ahelicopter of mass $1.0x10^3kg$ hovers by imparting a downward velocity v to the air displaced by its rotating blades. The area swept pout by the blades is $80m^2$. Calculate the value of v. (density of air=$1.3kgm^{-3}$)

   Solution

   $$F = \rho A v^2$$
   $$mg = \rho A v^2$$
   $$1.0x10^3x9.81 = 80xvx1.3x(v-0)$$
   $$1.0x10^3x9.81 = 104v^2$$
   $$v = 9.8m/s$$

4. Sand falls onto a conveyor belt at a constant rate of $2kgs^{-1}$. The belt is moving horizontally at $3ms^{-1}$. Calculate
   (a) The extra force required to maintain the speed of the belt
   (b) Rate at which this force is dong work
   (c) The rate at which the kinetic energy of the sand increases

   Solution

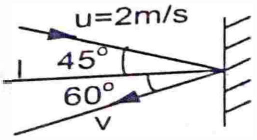
   Force =mass per second $x$velocity change
   $= 2x3 = 6N$

   Rate of doing work =force x velocity change
   $= 6x3 = 18Js^{-1}$

   Rate of k.e $= \frac{1}{2}m\ x\ (velocity\ change)^2$
   $= \frac{1}{2}x2x3^2 = 9Js^{-1}$

5. A ball of mass 0.25kg moving in a straight line with a speed of $2ms^{-1}$ strikes a vertical wall at an angle of 45° to the normal. The wall gives it an impulse in the direction of the normal and the ball rebounds at an angle of 60° to the normal. Calculate the magnitude of the impulse and the speed with which the ball rebounds.

   Solution



   Impulse $I = mv - mu$

   $$I = m\left[\begin{pmatrix}-vCos60\\-vSin60\end{pmatrix} - \begin{pmatrix}2cos45\\-2sin45\end{pmatrix}\right]$$

   $$I = \frac{1}{4}\left[\begin{pmatrix}\frac{-1}{2}v\\-\frac{\sqrt{3}}{2}v\end{pmatrix} - \begin{pmatrix}\sqrt{2}\\-\sqrt{2}\end{pmatrix}\right] = \frac{1}{4}\begin{pmatrix}-\frac{v}{2}-\sqrt{2}\\-\frac{\sqrt{3}}{2}v+\sqrt{2}\end{pmatrix}$$

   Since I is perpendicular to the wall then the vertical component is zero

   $$-\frac{v}{2} - \sqrt{2} = 0$$
   $$= -2\sqrt{2}m/s$$

   $$I = \frac{1}{4}\begin{pmatrix}-\frac{-2\sqrt{2}}{2}-\sqrt{2}\\-\frac{\sqrt{3}}{2}x-2\sqrt{2}+\sqrt{2}\end{pmatrix} = \frac{1}{4}\begin{pmatrix}0\\\sqrt{6}+\sqrt{2}\end{pmatrix} = \begin{pmatrix}0\\0.966\end{pmatrix}$$

   $$I = 0.966Ns$$

Exercise 14

1. A horizontal jet of water leaves the end of a hose pipe and strikes a wall horizontally with a velocity of 20m/s. If the end of the pipe has a diameter of 2cm, calculate the force that will be exerted on the wall. **An(125.7N)**
2. Water flows at 3m/s from a pipe of diameter of 0.1m and strikes a vertical plate near the outlet of the pipe. If the steam of water strikes the plate normally, calculate the force that will be exerted on the wall. **An(71N)**
3. Water emerges at $2ms^{-1}$ from a hose pipe and hits a wall at right angles. The pipe has cross-sectional area of $0.03m^2$. calculate the force on the wall assuming that the water does not rebound.(density of water $1000kgm^{-3}$) **An(120N)**
4. Water is squirting horizontally at $4.0ms^{-1}$ from a burst pipe at a rate of $3.0kgs^{-1}$. The water strikes a vertical wall at right angles and runs down it without rebounding. Calculate the force the water exerts on the wall **An(12N)**
5. A machine gun fires 300 bullets per minute horizontally with a velocity of $500ms^{-1}$. Find the force needed to prevent the gun moving backward if the mass of each bullet $8.0x10^{-3}kg$ **An(20N)**
6. Coal is falling onto a conveyor belt at a rate of 540 tones every hour. The belt is moving horizontally at $2.0ms^{-1}$. Find the extra force required to maintain the speed of the belt **An($3.0x10^2$N)**
7. A helicopter of total mass 1000kg is able to remain in a stationary position by imparting a uniform downward velocity to a cylinder of air below it of effective diameter 6m. assuming the density of air to be 1.2kgm, calculate the downward velocity given to air **An($17.2ms^{-1}$)**
8. (a) The rotating blades of a hovering helicopter seeps out an area of radius 4.0m imparting a down ward velocity of $12ms^{-1}$ to the air displaced. Find the mass of the helicopter.(density of air $1.3kgm^{-3}$) **An(940kg)**

   (b) the sped of rotation of the blades of the helicopter is now increased so that the air has a down ward velocity of $13ms^{-1}$. Find the upward acceleration of the helicopter **An($1.7ms^{-2}$)**
9. Find the force exerted on each square meter of a wall which is at right angles to a wind blowing at $20ms^{-1}$. Assume that the air does not rebound.(density of air $1.3kgm^{-3}$) **An(520N)**
10. Hail stones with an average mass of 4.0g fall vertically and strike a flat roof at $12ms^{-1}$. In a period of 5.0 minutes, 6000 hail stones fall on each square meter of roof and rebound vertically at $3.0ms^{-1}$. Calculate the force on the roof if it has an area of $30m^2$ **An(36N)**
11. A hose with a nozzle 80mm in diameter ejects a horizontal stream of water at a rate of $0.044m^3s^{-1}$.

    (a) With what velocity will the water leave the nozzle

    (b) What will be the force exerted on a vertical wall situated close to the nozzle and at right-angle to the stream of water, if after hitting the wall;

    (i) The water falls vertically to the ground

    (ii) The water rebounds horizontally

    An(8.75m/s, 385N, 770N)
12. An astronaut is outside her space capsule in a region where the effect of gravity can be neglected. She uses a gas gun to move herself relative to the capsule. The gas gun fires gas from a muzzle of area $1.60mm^2$ at a speed of $150ms^{-1}$. The density of the gas is $0.800kgm^{-3}$ and the mass of the astronaut including her space suit is 130kg. calculate

    (a) The mass of gas leaving the gun per second

    (b) The acceleration of the astronaut due to gun, assuming that the change in mass is negligible

    An($1.92x10^{-2}kgs^{-1}$, $2.22x10^{-2}ms^{-2}$)
13. Sand is poured at a steady rate of $5.0gs^{-1}$ on to the pan of a direct reading balance calibrated in grams. If the sand falls from a height of 0.20m on to the pan and it does not bounce off the pan then, neglecting any motion of the pan, calculate the reading on the balance 10s after the sand first hits the pan. **An(0.051kg)**
14. A top class tennis player can serve the ball, of mass 57g at an initial horizontal speed of 50m/s. the ball remains in contact with the racket for 0.050s. calculate the average force exerted on the ball during the serve **An(57N)**
15. A motor car collides with a crash barrier when travelling at 100km/h and is brought to rest in 0.1s.

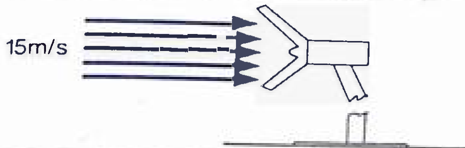
    (a) if the mass of the car and its occupants is 900kg calculate the average force on the car

16

(b) Because of the seat belt, the movement of the driver whose mass is 80 kg, is restricted to 0.20m relative tot the car. Calculate the average force exerted by the belt on the driver
An($2.5x10^5$N, $1.54x10^4$N)

16. A stone of mass 80kg is released at the top of a vertical cliff. After falling for by 3s, it reaches the foot of the cliff, and penetrates 9cm into the ground. What is;
    (a) The height of the cliff
    (b) The average force resisting penetration of the ground by the stone **An(45m, 400N)**

17. The blades of a large wind turbines, designed to generate electricity, sweeps pout an area of 1400m² and rotates about a horizontal axis which points directly into a wind of speed 15m/s

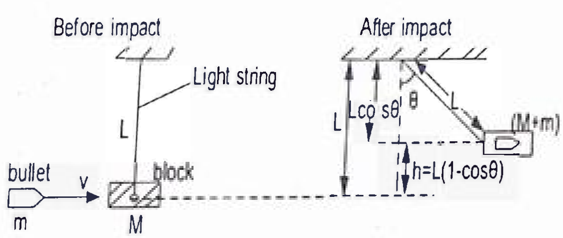


(a) Calculate the mass of air passing per second through the area swept out by the blades (take the density of air to be 1.2 kg/m³)
(b) The mean speed of the on the far side of the blades is reduced to 13m/s. how much kinetic energy is lost by the air per second **An($2.5x10^4$kg/s, $7.1x10^5$J/s))**

18. A ball of mas $6.0x10^{-2}$kg moving at $15ms^{-1}$ hits a wall at right angles and bounces off along the same line at $10ms^{-1}$
    (a) What is the magnitude of the impulse of the wall on the ball
    (b) The ball is estimated to be in contact with the wall for $3.0x10^{-2}$s, what is the average force on the ball
    An(1.5Ns,50N)

19. A body of mass 2.0kg and which is at rest is subjected to a force of 200N for 0.2s followed by a force of 400N for 0.30s acting in the same direction. Find
    (a) The total impulse on the body
    (b) The final speed of the body
    An(160Ns,80ms⁻¹)

4.1.9: BALLISTIC PENDULUM



Resolving along the vertical gives $Lcos\theta$
But $L = Lcos\theta + h$
$h = L - Lcos\theta = L(1 - cos\theta)$
The device illustrates the laws of conservation of momentum and mechanical energy

a) **During impact**
- Mechanical energy is not conserved because of friction and other non conservative forces
- Linear momentum is conserved in the horizontal direction along which there is no external force

If V_c is the velocity of combined mass just after collision
$$Mv + mx0 = (M + m)V_c$$
$$mv = (m + M)V_c \text{.......................................(i)}$$
The block was initially at rest.

b) **Swing after impact**
- Mechanical energy is conserved. The conserved gravitational force causes conversion of $k.e$ to $p.e.$
- Momentum is not conserved because an external resultant force (pull of the earth / weight) acts on the bullet-block system.

From (i) $k.e. = p.e.$
$$\frac{1}{2}(M + m)V_c^2 = (M + m)gh$$

But $h = L(1 - cos\theta)$
$$V_c^2 = 2gL(1 - cos\theta)$$
$$V_c = \sqrt{2gL(1 - cos\theta)} \text{.........(2)}$$
θ is the angle of swing

V_C =0.81ms^{-1}
The velocity of the composite just after collision is 0.81ms^{-1}

ii) Principle of mechanical energy at B

$K.E = P.E$

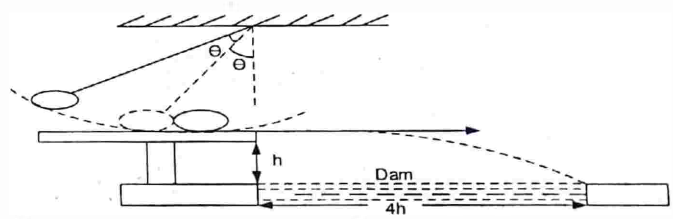
$\frac{1}{2}M_cV_c^2 = M_c gH$ but $M_c = (m + 2m)$

$H = \frac{1}{2}\frac{V_c^2}{g} = \frac{1}{2} x \frac{0.81^2}{9.81} = 0.33m$

iii) -Frictional force
-Air resistance

Exercise 15

1. A bullet of mass 40g is fired horizontally into freely suspended block of wood of mass 1.96kg attached at the end of an inelastic string of length 1.8m. Given that the bullet gets embedded in the block and the string is deflected through an angle of 60° to the vertical . Find:
   (i) The initial velocity of the bullet **An[210m/s]**
   (ii) The maximum velocity of the block. **An**[42m/s]
2. A bullet of mass 20g travelling horizontally at 100ms^{-1} embedded itself in the centre of a block of wood of mass 1kg which is suspended by a light vertical string 1m in length. Calculate the maximum inclination of the string to the vertical. **An(36.1°)**
3. A bullet of mass 50g travelling horizontally at 600ms^{-1} strikes a block of wood of mass 2kg which is suspended by a light vertical string so that its free to swing. The penetrates the block completely and emerges on the other side travelling at 400ms^{-1} in the same direction. As a result the block swings such that the string makes an angle of 25° with the horizontal. Calculate the length of the string. **An(1.719m)**
4. A block of wood of mass 1.00kg is suspended freely by a thread. A bullet of mass 10g is fired horizontally at the block and becomes embedded in it. The block swings to one suede rising a vertical distance of 50cm. with what speed did the bullet hit the block **An[319.4m/s]**
5. A bullet of mass 50g is fired horizontally into a block of wood of mass 8kg which is suspended by strings of length 2.5m long. After impact the block swings upwards through an angle of 30° to the vertical . Find the velocity of the bullet **An[32.7m/s]**
6. A simple pendulum consisting of a small heavy bob attached to a light string of length 40cm is released from rest with the string at 60° to the downward vertical. Find the speed of the pendulum bob as it passes through its lowest point **An(2.0ms^{-1})**
7. A circular ring is tied to a roof using a string of length, l and displaced such that it makes an angle of 2θ with the vertical, where $\theta = 30°$. It is then released to throw a spherical ball horizontally across the dam at a height, h. It collides in elastically with the ball when at angle θ and move together until the ball leaves the bench horizontally to cross the dam of width 4h.



if the bench is frictionless and the masses are equal, showthat $h = \frac{l(\sqrt{3}-1)}{32}$. Hence if $l = 128cm$,find the velociy with whoich the ball hits the ground

UNEB 2018 NO.1c

(i) Explain why a passenger in a car jerks forwards when the brakes are suddenly applied. (03 marks)

(*ii*) Use Newton's second law to define the Newton. (04 marks)

UNEB 2017 NO.1

(a) (i) State Newton's laws of motion (03marks)

(ii) A molecule of gas contained in a cube of side l strikes the wall of the cube repeatedly with a velocity u. Show that the average force F on the wall is given by $F = \frac{m u^2}{l}$ where m is the mass of the molecule (04marks)

(b) (i) Define the **linear momentum** and state the **law of conservation of linear momentum**. (02marks)