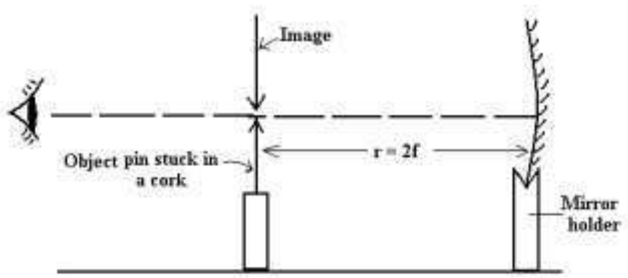
PHYSICS LIGHT NOTES CONTINUATION

4.0 Determination of the Focal Length of a Concave Mirror.

4.0.1 Method (1) Using a pin at C



An object pin is placed in front of a mounted concave mirror so that its tip lies along the axis of the mirror.

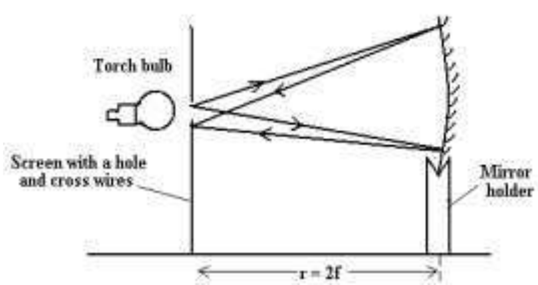
The position of the pin is adjusted until it coincides with its image such that there is no parallax between the pin and its image. The distance r of the pin from the mirror is measured. The required focal length

NOTE

(i) In the position where there is no parallax between the object pin and its image, there is no relative motion between the object and its image when the observer moves the head from side to side.

(ii) When the pin coincides with its image, the rays are incident normal to the mirror and are thus reflected along their own path. Therefore the pin coincides with its image at the Centre of curvature of the mirror.

4.0.2 Method (2) Using an illuminated object at C

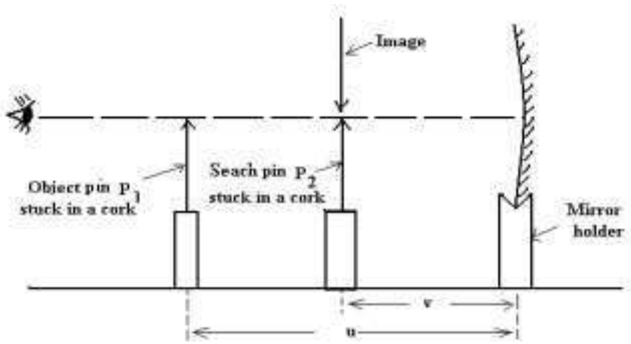


An illuminated bulb, a screen and a concave mirror mounted in a holder are aligned as shown above. The mirror position is adjusted to or from the screen until a sharp image of the cross-wire is formed on the screen besides the object.

The distance r of the mirror from the screen is measured.

The required focal length

4.0.3 Method (3) Using no parallax method in locating v



An object pin P_1 is placed at a distance u in front of a mounted concave mirror so that its tip lies along the axis of the mirror.

A search pin P_2 placed between the mirror and pin p_1 is adjusted until it coincides with the image of pin p_1 by no-parallax method.

The distance v of pin p_2 from the mirror is measured.

The procedure is repeated for several values of u and the results are tabulated including values of uv, and u+v.

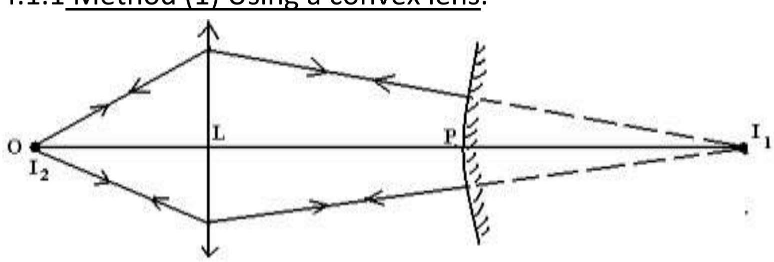
A graph of uv against u+v is plotted and the slope s of such a graph is equal to the focal length f of the mirror.

NOTE:

If a graph of $\frac{1}{u}$ against $\frac{1}{v}$ is plotted, then each intercept **c** of such a graph is equal to $\frac{1}{f}$. **Thus** $f = \frac{1}{c}$

1.1 Determination of Focal Length of a Convex Mirror.

4.1.1 Method (1) Using a convex lens.



An object O is placed in front of a convex lens to form a real image on the screen at I_1.

The distance LI_1 of the screen from the lens is then measured.

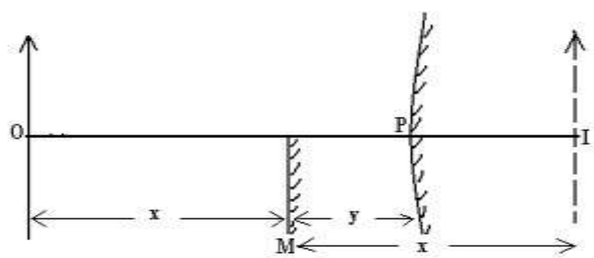
A convex mirror is placed between the lens and the screen and the mirror is then moved along the axis OI_1 until an image I_2 is formed besides O.

The distance LP is measured. The required focal length $f = \frac{PI_1}{2}$, where $PI_1 = LI_1 - LP$

NOTE ;

When the incident rays from an object are reflected back along the incident path, a real inverted image is formed besides the object in which case the rays strike the mirror normally. Therefore they will if produced pass through the Centre of curvature of the mirror thus distance PI_1 = radius of curvature

4.1.2 Method (2) Using No parallax.



An object pin O is placed in front of a convex mirror and a virtual diminished image is formed at I.

A plane mirror M is placed between O and P so as to intercept half the field of view of the convex mirror.

Mirror M is adjusted until its own image of O coincides with I by no parallax method. Measure the distances x and y.

The focal length of the mirror is the calculated from

Note:

(i) The two images coincides when they are as far behind the plane mirror as the object is in front.

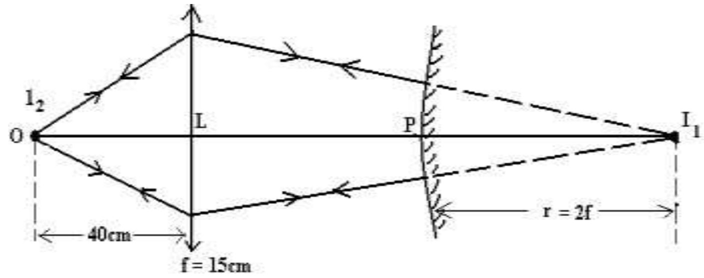
(ii) Substituting for u = x □ y and v = y □ x in the mirror formula gives

EXAMPLES

1. An object O is placed 40cm in front of a convex lens of focal length 15cm forming an image on the screen. A convex mirror situated 4cm from the lens in the region between the lens and the screen forms the final image besides object O. (i) Draw a ray diagram to show how the final image is formed.

(ii) Determine the focal length of the convex mirror.

Solution



Consider the action of a convex lens

u = 40cm, and f = 15cm, Using the lens formula $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ gives

$v = \frac{fu}{u - f} = \frac{15 \times 40}{40 - 15} = \mathbf{24cm}$, The radius of curvature r = (24 − 4) cm = 20cm

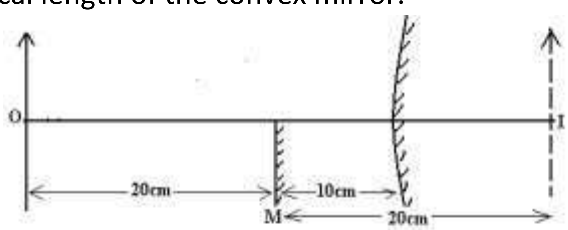
Using the relation r = 2f

∴ 2f = 20cm

∴ f = 10cm

Thus f = -10cm “The Centre of curvature of a convex mirror is virtual”

4. A plane mirror is placed 10cm in front of a convex mirror so that it covers about half of the mirror surface. A pin 20cm in front of the plane mirror gives an image in it, which coincides with that of the pin in the convex mirror. Find the focal length of the convex mirror.



Consider the action of a convex mirror

u = 30cm and v = − (20 − 10) = −10cm “The image formed is virtual “

Using the mirror formula gives,

EXERCISE

1. Describe an experiment to determine the focal length of a concave mirror.
2. You are provided with the following pieces of apparatus: A screen with cross wires, a lamp, a concave mirror, and a meter ruler. Describe an experiment to determine the focal length of a concave mirror using the above apparatus.
3. Describe an experiment, including a graphical analysis of the results to determine the focal length of a concave mirror using a no parallax method.

4. Describe an experiment to measure the focal length of a convex mirror
5. Describe how the focal length of a diverging mirror can be determined using a convex lens.
6. Describe how the focal length of a convex mirror can be obtained using a plane mirror and the no parallax method.
7. A plane mirror is placed at a distance d in front of a convex mirror of focal length f such that it covers about half of the mirror surface. A pin placed at a distance L in front of the plane mirror gives an image in it, which coincides with that of the pin in the convex mirror. With the aid of an illustration, Show that $2df = d^2 \square L^2$

5.0 REFRACTION OF LIGHT

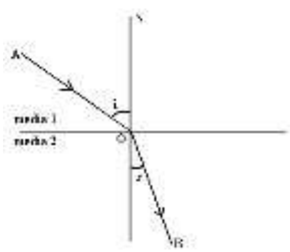
Refraction is the bending of light rays at an interface between two media of different optical densities

NOTE ;

Light is refracted because it has different speeds in different media.

5.0.1 Laws of Refraction

Consider a ray of light incident on an interface between two media as shown.



O = Point of incidence.

AO = Incident ray

OB = Refracted ray.

ON = Normal at O

$\square i$ =Angle of incidence

$\square r$ =Angle of refraction

LAW 1:

The incident ray, the refracted ray and the normal at the point of incidence all lie in the same plane.

LAW 2:

The ratio of the sine of the angle of incidence to the sine of the angle of refraction is a constant for a given pair of media of different optical densities Thus

REFRACTIVE INDEX. [n] is the ratio of the sine of angle of incident to the sine of angle of refraction for a ray of light traveling from air in to a given medium.

OR

Is the ratio of the speed of light in a vacuum to speed of light in a medium.

Thus Refractive index, n = speed of light in a vacuum, c / speed of light in a medium, c_m

Where speed of light in a vacuum $c = 3\square 0 \square 10^8$ $ms^{\square 1}$.

NOTE:

The refractive index, n for a vacuum is 1. However if light travels from air to another medium, the value of n is slightly greater than 1.For example, $n = 1\cdot33$ for water and $n = 1\cdot5$ for glass.

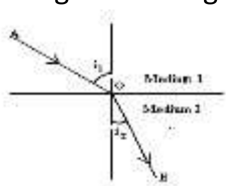
THE PRINCIPLE OF REVERSIBILITY OF LIGHT.

It states that the paths of light rays are reversible. This means that a ray of light can travel from medium 1 to 2 and from 2 to 1 along the same path.

RELATIONS BETWEEN REFRACTIVE INDICES

CASE I:

Consider a ray of light traveling from medium 1 (air) to medium 2 (glass) as shown.



Suppose i_1 and i_2 are the angles of incidence and refraction respectively in medium 1 and medium 2, then

For light traveling from (1) to (2), refractive index $_1n_2 = \frac{\sin i_1}{\sin i_2}$ ------------------(i)

For light traveling from (2) to (1), refractive index $_2n_1 = \frac{\sin i_2}{\sin i_1}$

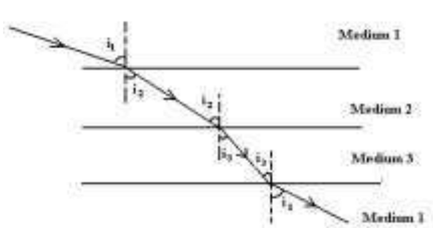
$$\Rightarrow \frac{1}{_2n_1} = \frac{\sin i_1}{\sin i_2}$$ ---------------------------(ii) (reciprocal of the above equation)

Equating equation (i) and (ii) gives

$$_1n_2 = \frac{1}{_2n_1} \quad \text{OR} \quad _1n_2 \times {_2n_1} = 1$$

CASE II:

Consider a ray of light moving from medium 1 (air) through a series of media 2,3 and then finally emerge into medium 1 (air) as shown.



At medium 2-medium 3 interface, Snell's law gives.

$$_2n_3 = \frac{\sin i_2}{\sin i_3} = \frac{\sin i_2}{\sin i_1} \times \frac{\sin i_1}{\sin i_3} = {_2n_1} \times {_1n_3}$$

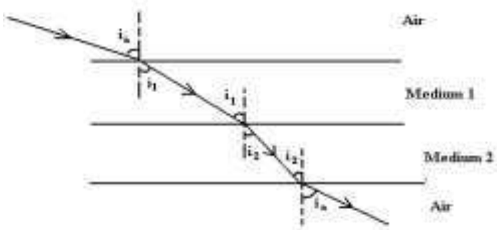
$\Rightarrow \; _2n_3 = {_2n_1} \times {_1n_3}$

Using the relation $_2n_1 = \frac{1}{_1n_2}$

$\Rightarrow \; _1n_3 = {_1n_2} \times {_2n_3}$ OR $_2n_3 = \frac{_1n_3}{_1n_2}$

GENERAL RELATION BETWEEN n AND sin i

Consider a ray of light moving from air through a series of media 1, 2 and then finally emerge into air as shown.



At air – medium 1 interface, Snell's gives $\frac{\sin i_a}{\sin i_1} = n_1$

$\Rightarrow \; \sin i_a = n_1 \sin i_1$ -- (i)

At air - medium 2 interface, Snell's gives $\frac{\sin i_a}{\sin i_2} = n_2$

$\Rightarrow \; \sin i_a = n_2 \sin i_2$ --------------------------------------(ii)

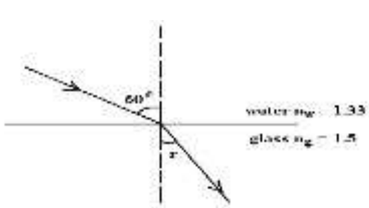
Equating equation (i) and (ii) gives

$n_1 \sin i_1 = n_2 \sin i_2$.

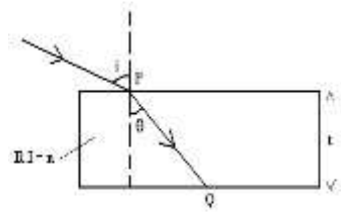
$\Rightarrow$ n sin i = a constant.

EXAMPLES:

1. Monochromatic light incident on a block of material placed in a vacuum is refracted through an angle θ. If the block has a refractive index n and is of thickness t, show that this light takes a time $\frac{n t \sec\theta}{c}$ to emerge from the block. where c is the speed of light in a vacuum.



Solution:



Let T be the time taken by light to travel from point P to Q in the medium.

Thus $T = \dfrac{\text{distance PQ}}{\text{speed of light in the block, } c_m}$

Where distance $PQ = \dfrac{t}{\cos\theta} = t\sec\theta$

$\therefore \quad T = \dfrac{t\sec\theta}{c_m}$ ------------------------------------(i)

By definition, $n = \dfrac{c}{c_m}$ $\quad \therefore\ c_m = \dfrac{c}{n}$ ---------------(ii)

Substituting equation (ii) in to (i) gives

$T = \dfrac{n\,t\sec\theta}{c}$

2. A monochromatic beam of light is incident at 60° on a water-glass interface of refractive index 1.33 and 1.5 respectively as shown

Calculate the angle of reflection r.

Solution:

Applying Snell's law

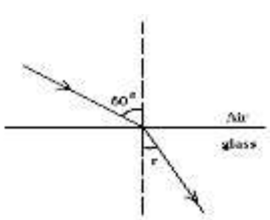
gives $n_w \sin 60° = n_g \sin r$ 1.33

$\sin 60° = 1.5 \sin r$

Thus $\therefore\ r = 50.2°$.

3. A ray of light propagating from air is incident on an air-glass interface at an angle of 60°. If the refractive index of glass is 1.5, calculate the resulting angle of refraction.

Solution.



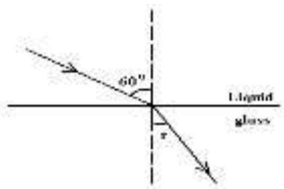
Applying Snell's law gives

$n_a \sin 60° = n_g \sin r$ but $n_a = 1$, $n_g = 1.5$

$\Rightarrow$ $1 \sin 60° = 1.5 \sin r$

$\Rightarrow$ $r = 35.6°$.

4. A monochromatic ray of light is incident from a liquid on to the upper surface of a transparent glass block as shown.



Given that the speed of light in the liquid and glass is 2.4×10^8 ms^{-1} and 1.92×10^8 ms^{-1} respectively, find the angle of refraction, r.

Solution:

Applying Snell's law gives $n_l \sin 60° = n_g \sin r$

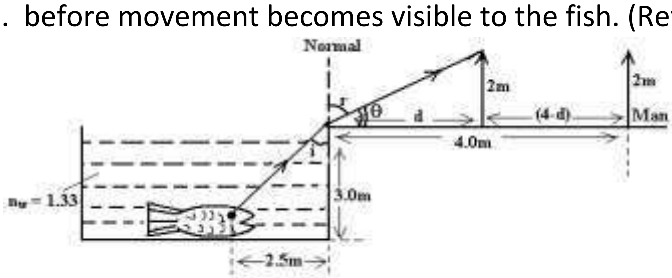
$\frac{c}{c_l} \sin 60° = \frac{c}{c_g} \sin r$

$\Rightarrow$ $\sin r = \frac{c_g}{c_l} \sin 60°$ but $c_g = 1.92 \times 10^8$ ms^{-1} , $c_l = 2.4 \times 10^8$ ms^{-1}

$\Rightarrow$ $\sin r = \frac{1.92 \times 10^8}{2.4 \times 10^8} \sin 60°$

$\Rightarrow$ $r = 43.9°$.

5. A small fish is 3.0m below the surface of the pond and 2.5m from the bank. A man 2.0m tall stands 4.0m from the pond. Assuming that the sides of the pond are vertical, calculate the distance the man should move towards the edge of the pond . before movement becomes visible to the fish. (Refractive index of water = 1.33).



From the diagram, $\tan i = \frac{2.5}{3}$ $\Rightarrow$ $i = 39.81°$

Applying Snell's law at the edge of the pond gives

$n_w \sin i = n_a \sin r$

1□33 sin 39□81□ = 1 sin r

□ □ r = 58□4□

Thus □□ = 90□ □ 58□4□ = 31□6□

From the diagram tan □ = 2 / d   □ d = 2 / tan 31□6□   □ d = 3□2m.

Thus required distance traveled = 4 □ d

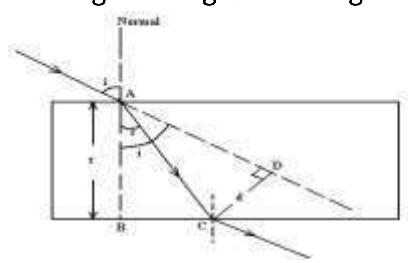
= 4□ 3□2

= 0□8m

SIDE WISE DISPLACEMENT OF LIGHT RAYS.

When light travels from one medium to another, its direction is displaced side ways. This is called lateral displacement.

Consider a ray of light incident at an angle i on the upper surface of a glass block of thickness t, and then suddenly refracted through an angle r causing it to suffer a sidewise displacement d.



From □ ABC, AC = t / cos r --------------------------------.(i)

From the diagram, □ CAD = (i □ r)

From □ ACD, AC = d / sin (i □ r) -----------------------(ii)

Equating equation (i) and (ii) gives

t / cos r = d / sin (i □ r)

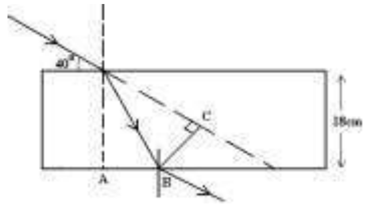
□ d = t sin (i□r) / cos r

NOTE :

The horizontal displacement of the incident ray , BC = t. tan r

Example:

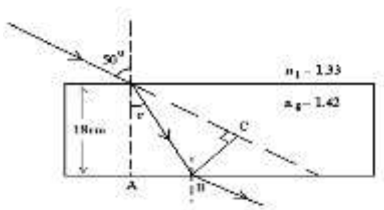
The figure below shows a monochromatic ray of light incident from a liquid of refractive index 1□33 onto the upper surface of a glass block of refractive index 1□42



Calculate the;

(i) horizontal displacement AB.

(ii) lateral displacement BC of the emergent light. Solution



(i) Applying Snell's law at the liquid- glass interface gives, n_l sin 50□ = n_g sin r.

1□33 sin 50□ = 1□42 sin r

□ □r = 45□8□

Horizontal displacement AB = t tan r

= 18 tan 45□8□

= 18□51cm

(ii) Lateral displacement d = $\frac{\text{t sin (i□r)}}{\text{cos r}}$

d = $\frac{\text{18 sin (50□ □ 45□8□)}}{\text{cos 45□8□}}$.

d = 1□89cm.

EXERCISE1

(1) What is meant by refraction of light?

(2) (i) State the laws of refraction of light.

(ii) State what brings about refraction of light as it travels from one medium to another.

(3) (i) What is meant by the refractive index of a material?

(ii) Light of two colours blue and red is incident at an angle □ from air to a glass block of thickness t. When blue and red lights are refracted through angles of θ_b and θ_r respectively, their corresponding speeds in the glass block are

v_b and v_r . Show that the separation of the two colours at the bottom of the glass block t v^r - v^b

 sin γ Where θ_r θ_b and c is the speed of light in air.

d cos θ_r cos θ_b c

(iii) Light consisting of blue and red is incident at an angle of 60 from air to a glass block of thickness 18cm. If the speeds of blue and red light in the glass block are $1.86 \times 10^8 ms^{-1}$ and $1.92 \times 10^8 ms^{-1}$ respectively, find the separation of the two colours at the bottom of the glass block..

Answer: 0.54cm

(4) Show that when the ray of light passes through different media separated by plane boundaries, n sin = constant where n is the absolute refractive index of a medium and is the angle made by the ray with the normal in the medium.

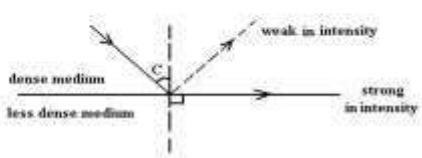
(5) Show that when the ray of light passes through different media 1 and 2 separated by plane boundaries, $_1n_2 \times {}_2n_1 = 1$ where n is the refractive index of amedium.

(6). Show that when the ray of light passes through different media 1,2 and separated by plane boundaries, $_1n_3 = {}_1n_2 \times {}_2n_3$ where n is the refractive index of a medium.

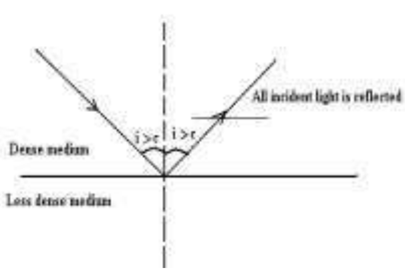
(7) Show that a ray of light passing through a glass block with parallel sides of thickness t suffers a sidewise displacement $d = \frac{t \sin(\phi - \lambda)}{\cos \lambda}$ where is the angle of incidence and is the angle of refraction.

CRITICAL ANGLE AND TOTAL INTERNAL REFLECTION

Consider a ray of light incident on an interface between two media as shown.



critical angle: is the angle of incidence in the dense medium which results into an angle of refraction of 90 in the adjoining less dense medium. If the angle of incidence is greater than the critical angle, all the incident light energy is reflected back in the dense medium and total internal refection is said to have occurred.



NOTE :

(a) At critical point Snell's law becomes.

$n_1 \sin c = n_2 \sin 90^\circ$

$\therefore \quad \sin c = \frac{n_2}{n_1}$

$\therefore \quad c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$

Where n_1 and n_2 are the refractive indices of the dense and the less dense medium respectively.
However if the less dens medium is air, then Snell's law becomes

$n_1 \sin c = n_a \sin 90^\circ$

$\therefore \quad \sin c = \frac{1}{n_1}.$

(b) The conditions for total internal reflection to occur are;

(i) Light should travel from an optically denser to a less dense medium.

(ii) The angle of incidence at the boundary of the media should be greater than the critical angle.

APPLICATION OF TOTAL INTERNAL REFLECTION.

(i) It is responsible for the formation of a mirage.

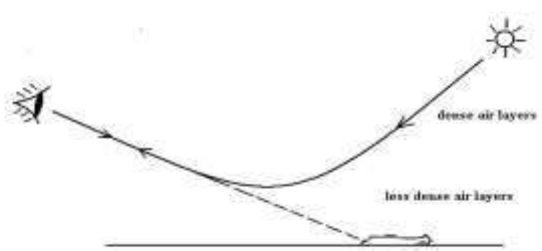
(ii) It is responsible for the formation of a rainbow.

(iii) It is responsible for the transmission of light in optical fibres.

(iv). It is responsible for the transmission of sky radio waves

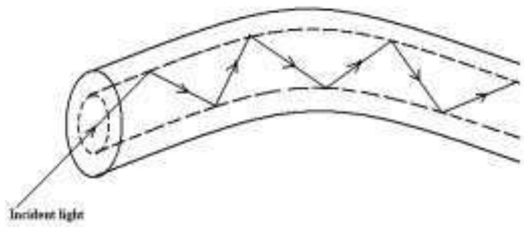
(v). It is responsible for the transmission of light in prism binoculars.

FORMATION OF A MIRAGE



On a hot day, The air layers near the earth's surface are hot and are less denser than the air layers above the earth's surface. Therefore as light from the sky pass through the various layers of air, light rays are continually refracted away from the normal till some point where light is totally internally reflected. An observer on earth receiving the totally internally reflected light gets an impression of a pool of water on the ground and this is the virtual image of the sky.

AN OPTICAL FIBRE



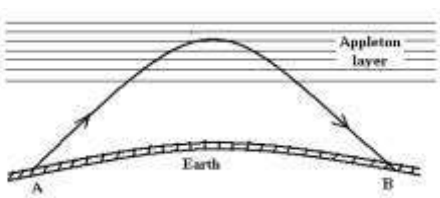
When a light ray enters in to the fibre, it bounces from one edge to another by total internal reflection. These successive total internal reflections enable the transmission of light in optical fibres.

NOTE

(i) An optical fibre finds a practical application in an endoscope, a device used by doctors to inside the human body.

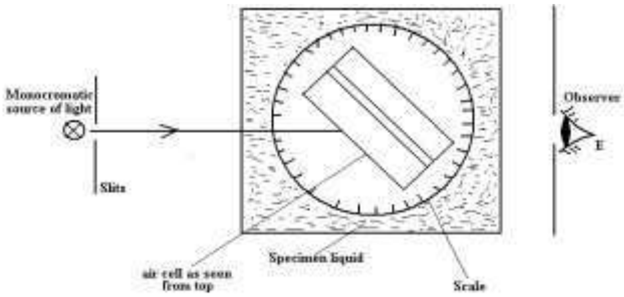
(ii) Optical fibres are used in telecommunication systems (i.e. Telephone or TV signals are carried along optical fibers by laser light).

SKY RADIO WAVES



A radio wave sent skyward from a station transmitter A is continually refracted away from the normal on entering the electron layer that exists above the earth's surface. Within the electron layer, the wave is totally internally reflected causing it to emerge from the electron layers and finally returns to the earth's surface where it's presence can be detected by a radio receiver at B

DETERMINATION OF REFRACTIVE INDEX OF A LIQUID BY AN AIR-CELL METHOD

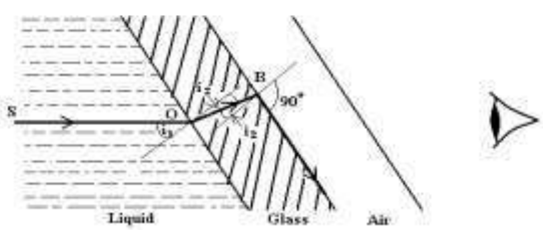


The air cell is immersed in a liquid under test. A beam of monochromatic light is directed onto the air cell and then observed through the cell from the opposite side at E. The cell is then rotated on one side until light is suddenly cut off and the angular position θ_1 is noted. The cell is again rotated in the opposite direction until light is suddenly cut off and the angular position θ_2 is noted. The refractive index of the liquid can then be calculated from $n = \frac{1}{\sin\theta}$ Where $\theta = \frac{\theta_1 + \theta_2}{2}$

NB:

The source of light should be monochromatic so that the extinction of light is sharp since monochromatic light does not under go dispersion, as it is with white light.

THEORY OF THE AIR-CELL METHOD



Ray OS is refracted along OB in glass. However, at B total internal just begins.

Suppose i_1 is the angle of incidence in the liquid, i_2 is the angle of incidence in the glass while n and n_g are the corresponding refractive indices, Then applying the relation n sin i = a constant gives $n \sin i_1 = n_g \sin i_2 = n_a \sin 90°$

$\therefore n \sin i_1 = n_a \sin 90°$

$\therefore n \sin i_1 = 1$

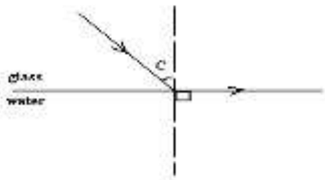
Thus $n = \frac{1}{\sin i_1}$

But $\therefore i_1 = \frac{\theta_1 + \theta_2}{2}$ Hence $n = \frac{1}{\sin\theta}$

Examples:

1 The critical angle for water-air interface is 48˚ 42[1] and that of glass-air interface is 38˚ 47[1]. Calculate the critical angle for glass-air interface.

Solution



Applying Snell's law gives n_g sin c = n_w sin 90º--------------------(i).

Given that for water-air interface c_w = 48º42[1].

□ n_w sin c_w = 1

n_w sin 48º42[1] = 1

□ n_w = 1□33---(ii)

Also for glass-air interface c_g = 38º47[1]

n_g sin c_g = 1

□ n_g sin 38º47[1] = 1

□ n_g = 1□67--(iii)

Substituting equation (ii) and (iii) into equation (i) gives

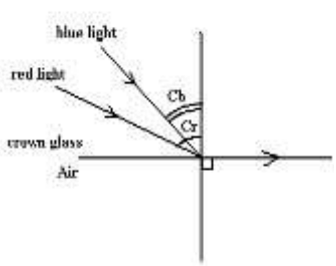
1□67sin c = 1□33 sin 90□

□ c = 52□8□

2. The refractive index for red light is 1□634 of crown glass and the difference between the critical angles of red and blue light at the glass-air interface is 0□56[1].What is the refractive index of crown glass for blue light

Solution

Analysis the critical angle between two media for red light is greater than that for any other light colour. This gives rise to the ray diagram below



Since c_r □ c_b , then c_r □ c_b = 0□56[1] ----------------------------(i)

Applying Snell's law to red light gives

n_r sin c_r = 1 1□63 sin c_r = 1

□ c_r = 37□73□

Equation (i) now becomes

c_b = c_r □ 0□56[1]

c_b = 37□73□ □ 0□56[1]

c_b = 36□8□

Applying Snell's law to blue light gives

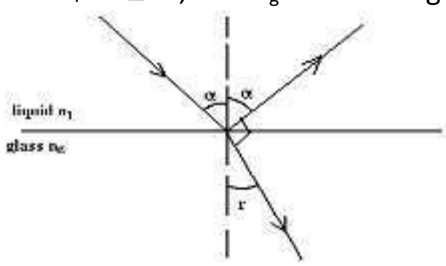
n_b sin c_b = 1

n_b sin 36□8□ = 1

□ n_b =1□67

2. (i) A glass block of refractive index n_g is immersed in a liquid of refractive index n_l. A ray of light is partially reflected and refracted at the interface such that the angle between the reflected and the refracted ray is 90□. Show that n_g = n_l tan □ where □ is the angle of incidence from the liquid to glass.

(ii) When the procedures in (i) above are repeated with the liquid removed, the angle of incidence increases by 8□. Given that n_l = 1□33, find n_g and the angle ofincidence at the liquid-glass interface. Solution



Applying Snell's law at the liquid-glass interface gives

n_g sinr = n_l sin □

But r + 90□+ □ = 180□

□ r = 90□ □ □

□ n_g sin (90□ □ □) □ n_l sin □.

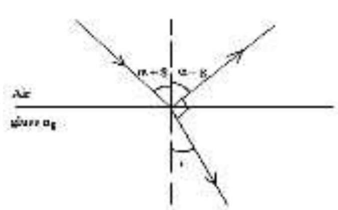
From trigonometry, sin (90□ □ □) = cos □

□ n_g cos □ = n_l sin □.

Dividing cos □ on both sides gives

n_g = n_l tan □.

(ii) When the liquid is removed.



From $n_g = n_l$ tan □---(i)

□ $n_g = n_a$ tan (□ + 8□) but $n_a = 1$

□ n_g = tan □ + tan 8□ . 1□ tan

□ tan 8□

n_g (1□ tan □ tan 8□) = tan □ + tan 8□

n_g □ n_g tan □ tan 8□ = tan □ + tan 8□---------------------------------(ii)

from equation (i) tan □ = $\frac{n_g}{n_l}$

Substituting for tan □ in equation (ii) gives

n_g □ $\frac{n^2_g \tan 8□}{n_l}$ = $\frac{n_g}{n_l}$ + tan 8□

but n_l = 1□33.

□ n^2_g □ 2.340 n_g + 1□326 = 0 ---------------------------

(iii) Equation (iii) is quadratic in n_g and solving it gives n_g = 1□39 or n_g not physically possible. Using equation (i)

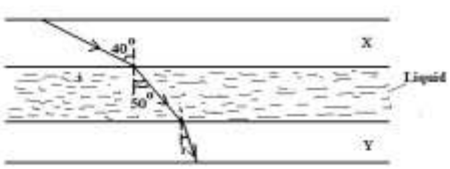
tan □ = $\frac{ng}{n_l}$ = $\frac{1□39}{1□33}$

□ = tan $^{□1}$ (1.045) = 46□3□

The required angle of incidence = □ + 8□

= 54□3□

3. The figure below shows a liquid layer confined between two transparent plates X and Y of refractive index 1□54 and 1□44 respectively.



A ray of monochromatic light making an angle of 40□ with the normal to the interface between media X and the liquid is refracted through an angle of 50□ by the liquid. Find the (i) refractive index of the liquid.

(ii) angle of refraction ,r in the medium Y.

(iii) minimum angle of incidence in the medium X for which the light will not emerge from medium Y.

Solution

(i) Applying Snell's law at the plate X – liquid interface gives n_x sin 40□ = n_l sin 50□

1□54 sin 40□ = n_l sin 50□ n_l = $\frac{1□54 \sin 40□}{\sin 50□}$

$\therefore\ n_l = 1.29$

(ii) Applying Snell's law at the liquid – plate Y interface

gives $n_l \sin 50° = n_y \sin r$

$1.29 \sin 50° = 1.44 \sin r$

$\therefore\ r = 43.3°$

(iii) For light not to emerge from plate Y, it grazes the liquid – plate Y interface.

$\therefore\ r = 90°$

Applying Snell's law at the liquid – plate Y interface

gives $n_l \sin i_l = n_y \sin 90°$

$1.29 \sin i_l = 1.44 \sin 90°$

$\sin i_l = \frac{1.44}{1.29}$ --------------------------------(i)

More over, applying Snell's law at the plate X – liquid interface gives

$n_x \sin i_x = n_l \sin i_l$

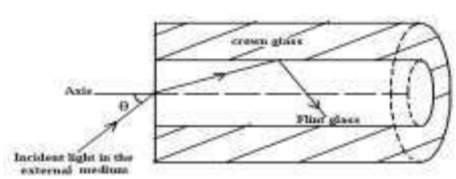
$1.54 \sin i_x = 1.29 \sin i_l$ ----------------------------(ii)

Substituting equation (i) in (ii) gives

$1.54 \sin i_x = 1.29 \times \frac{1.44}{1.29}$

$\therefore\ i_x = 40.5°$

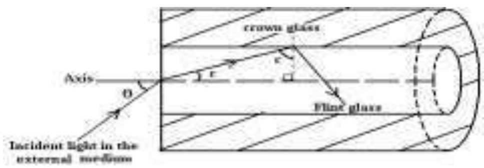
5.The diagram below shows a cross-section through the diameter of the light pipe with an incident ray of light in its plane.



The refractive indices for flint glass, crown glass and the external medium are n_1, n_2 and n_3 respectively. Show that a ray that enters the pipe is totally reflected at the flint -crown glass interface provided $\sin\theta \leq \frac{\sqrt{n_1^2 - n_2^2}}{n_3}$ where θ is the maximum angle of incidence in the external medium.

Solution

Analysis for light to be totally reflected, it must be incident at a critical angle on the flint-crown glass interface



Applying Snell's law at the external medium-flint glass interface gives

$n_3 \sin\theta = n_1 \sin r$ but $r + c = 90°$ $\Rightarrow$ $r = 90° - c$

$\Rightarrow$ $n_3 \sin\theta = n_1 \sin(90° - c)$

$\Rightarrow$ $n_3 \sin\theta = n_1 \cos c$

$\Rightarrow$ $\cos c = \frac{n_3 \sin\theta}{n_1}$ --------------------------------(i)

Applying Snell's law at the flint-crown glass interface gives

$n_1 \sin c = n_2 \sin 90°$

$\Rightarrow$ $\sin c = \frac{n_2}{n_1}$ ---(ii)

Using the trigonometrical relation $\sin^2 c + \cos^2 c = 1$, then

$$\left(\frac{n_2}{n_1}\right)^2 + \left(\frac{n_3 \sin\theta}{n_1}\right)^2 = 1$$

Thus, $\sin\theta = \frac{\sqrt{n_1^2 - n_2^2}}{n_3}$

EXERCISE 2

1. Explain the term total internal reflection and give three instances where it is applied.
2. With the aid of suitable ray diagrams, explain the terms critical angle and total internal reflection.
3. Show that the relation between the refractive index n of a medium and critical angle c for a ray of light traveling from the medium to air is given by $n = \frac{1}{\sin c}$

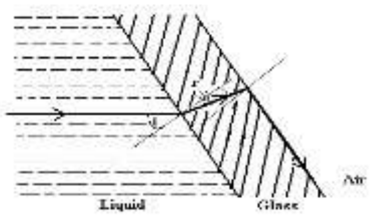
4.Show that the critical angle, c at a boundary between two media when light travels from medium 1 to medium 2 is given by $\sin c = \frac{n_2}{n_1}$ where n_1 and n_2 are the refractive indices of the media respectively.

5. Explain how a mirage is formed.

6.Explain briefly how sky radio waves travel from a transmitting station to a receiver.

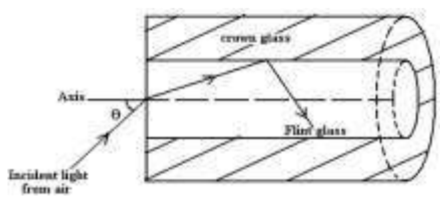
7. Describe how you would determine the refractive index of the liquid using an air cell.

8)



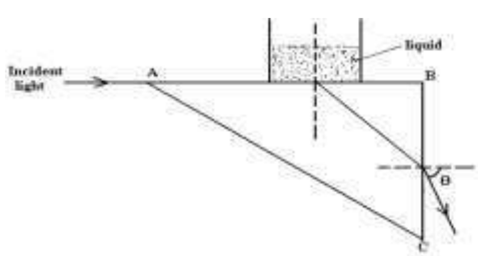
In the figure above, a parallel sided glass slide is in contact with a liquid on one side and air on the other side. A ray of light incident on glass slide from the liquid emerges in air along the glass-air interface. Derive an expression for the absolute refractive index ,n, of the liquid in terms of the angle of incidence i the liquid-medium.

(9) The diagram below shows a cross-section through the diameter of the light pipe with an incident ray of light in its plane.



The refractive indices for flint glass and crown glass are n_1 and n_2 respectively. Show that a ray which enters the pipe is totally reflected at the flint-crown glass interface provided $\sin\theta$ □ $\sqrt{n_1^2 - n_2^2}$ where θ is the maximum angle of incidence at the air-flint glass interface

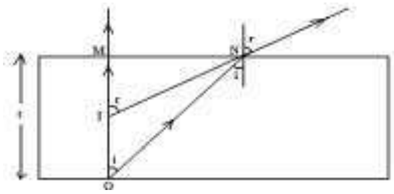
10.A liquid of refractive index □$_L$ is tapped in contact with the base of a right-angled prism of refractive index □$_g$ by means of a transparent cylindrical pipe as shown.



Show that a ray of light which is at a grazing incidence on the liquid-glass interface emerges in to air through face BC at an angle □ below the horizontal provided $\sqrt{\mu_L}$ □ $\mu_g^2 - \sin^2\theta$. Hence find □$_L$, if □$_g$ = 1□52 and □ =47□4□

REAL AND APPARENT DEPTH

Consider an object O viewed normally from above through a parallel-sided glass block of refractive index n_g and thickness t as shown.



A ray from an object O normal to the glass surface at M passes un deviated. While the ray ON inclined at a small angle i to the normal is refracted at N. The observer above the glass block sees the image of the object O at I.

Applying Snell's law at C gives

$n_g \sin i = n_a \sin r$------------------------------------(i).

From the diagram, $\sin i = \frac{MN}{ON}$ and $\sin r = \frac{MN}{IN}$

Equation (i) now becomes $n_g \frac{MN}{ON} = \frac{MN}{IN}$ ⇒

$n_g = \frac{ON}{IN}$

Since angle i is very small, then ON ≈ OM and IN ≈ IM

⇒ $n_g = \frac{OM}{IM}$

From the diagram, OM = real depth and IM = apparent depth.

Hence $n_g = \frac{\text{real depth}}{\text{apparent depth}}$.

If the apparent displacement of the object OI = d, then IM = (t − d).

So that $n_g = \frac{OM}{IM} = \frac{t}{t - d}$

Thus apparent displacement $d = \left(1 - \frac{1}{n_g}\right) t$

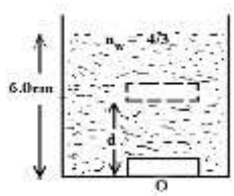
NOTE

(i) The apparent displacement d of an object O is independent of the position of O below the glass block. Thus the same expression above gives the displacement of an object which is some distance in air below a parallel-sided glass block.

(ii) If there are different layers of different transparent materials resting on top of each other, the apparent position of the object at the bottom can be found by adding the separate displacements due to each layer.

EXAMPLES:

1. An object at a depth of 60cm below the surface of water of refractive index 4/ is observed directly from 3 above the water surface. Calculate the apparent displacementof the object



— 1 - 1 t gives

Using the relationd

 ng

  3

d 1 - _ 6

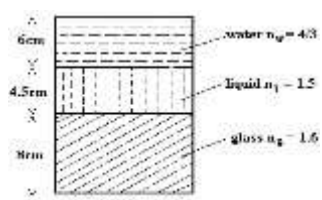
  4

d = 15 cm

2. A tank contains a slab of glass 8cm and refractive index 16. Above this is a depth of 45cm of a liquid of refractive index 15 and upon this floats 6cm of water of refractive index 4 calculate the apparent displacement of an object at the bottom of 3

the tank to an observer looking down wards directly from above.

Solution



Apparent displacement d = d_w + d_l + d_g

  1

Using the relation d 1 - _ t

  n

1 — 1 — — 1 1 t_g

 d 1 n_w t_w 1 n_l t_l

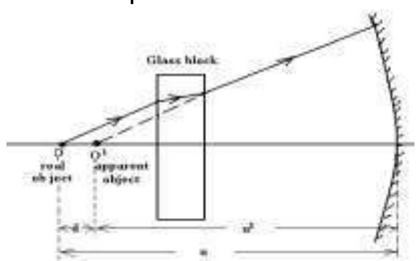
   n_g

$$\therefore \quad d = \left(1 - \frac{3}{4}\right)6 + \left(1 - \frac{1}{1.5}\right)4.5 + \left(1 - \frac{1}{1.6}\right)8$$

d = 1.5 + 1.5 + 3

d = 6cm.

3.A small object is placed 20cm in front of a concave mirror of focal length 15cm.
A parallel-sided glass block of thickness 6cm and refractive index 1.5 is then placed between the mirror and the object. Find the shift in the position and size of the image



Consider the action of a concave mirror in the absence of a glass block

u = 20cm and f = 15cm

using the mirror formula $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ gives

$$v = \frac{fu}{u - f} = \frac{15 \times 20}{20 - 15} = 60\text{cm}$$

Thus in the absence of a glass block, image distance = 60cm

In this case, magnification $m = \frac{v}{u} = \frac{60}{20} = 3$

Consider the action of a glass block

Apparent displacement $d = \left(1 - \frac{1}{n_g}\right)t = \left(1 - \frac{1}{1.5}\right) \times 6 = 2\text{cm}$

Thus in the presence of a glass block, object distance u¹ = (20 – 2) cm =18cm

"The object is displaced and it appears to be 18cm in front of the mirror" Consider the action of a concave mirror in the presence of a glass block u¹ = 18cm and f = 15cm

using the mirror formula $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ gives

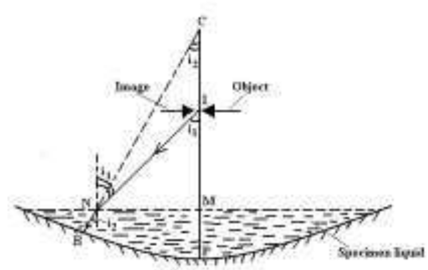
1

$$v^1 = \frac{fu^1}{u^1 - f} = \frac{15 \times 18}{18 - 15} = 90\text{cm}$$

$\therefore$ The required shift in the image position = $v^1 - v$ = (90 – 60) cm = 30cm

The magnification now becomes $m^1 = \frac{v^1}{u^1} = \frac{90}{18} = 5$

DETERMINATION OF REFRACTIVE INDEX OF A LIQUID USING A CONCAVE MIRROR METHOD.



A small quantity of the liquid under test is poured into a concave mirror of known radius of curvature r. An object pin is moved along the principal axis of the mirror until it coincides with its image at I. The distance IP is noted. The required refractive index $n_l = \frac{r}{IP}$.

PROOF

For refraction at N, $n_l \sin i_2 = n_a \sin i_1$ --------------------------- (i)

From the diagram, $\sin i_2 = \frac{NM}{NC}$ and $\sin i_1 = \frac{MN}{NI}$

Equation (i) now becomes

$$n_1 \frac{NM}{NC} = \frac{NM}{NI}.$$

On simplifying, $n_l = \frac{NC}{NI}$

But N is very close to M hence NC $\approx$ MC and NI $\approx$ MN

$$\therefore \quad n_l = \frac{MC}{MI}$$

Also for a small quantity of the liquid, M is close to P $\therefore$ MC $\approx$ CP = r, and MI $\approx$ IP.

Thus $n_l = \frac{r}{IP}$.

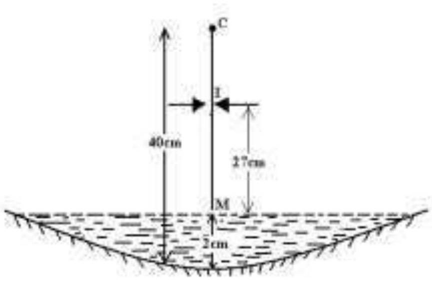
NOTE :

(i) If the specimen liquid is of reasonable quantity, then its depth d can not be ignored. In this case, $n_l = \frac{MC}{MI} = \frac{r-d}{MI}$

(ii) if the radius of curvature r of the concave mirror is not known, first determine it using the method discussed in the previous section.

EXAMPLES:

1.A liquid is poured in to a concave mirror to a depth of 2·0cm. An object held above the liquid coincides with its own image when its 27·0cm above the liquid surface. If the radius of curvature of the mirror is 40·0cm, calculate the refractive index of the liquid.

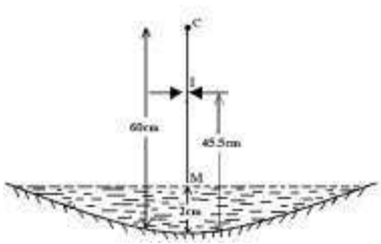


Using the relation $n_l = \frac{r-d}{MI}$

$= \frac{40 - 2}{27}$

$\therefore \quad n_l = 1·4$

2. A liquid is poured into a concave mirror to a depth of 2·0cm. An object held above the liquid coincides with its image when it is 45·5cm from the pole of the mirror. If the radius of curvature of the mirror is 60·0cm, calculate the refractive index of the liquid.



Using the relation $n_l = \frac{r-d}{MI}$

$= \frac{60 - 2}{45·5 - 2}$

$\therefore \quad n_l = 1·33$

3. A small concave mirror of focal length 8cm lies on a bench and a pin is moved vertically above it .At what point will the pin coincide with its image if the mirror is filled with water of refractive index 4/3. Solution ANALYSIS

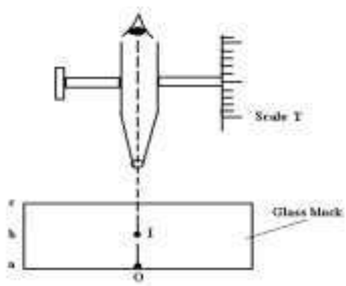
For a small concave mirror, the quantity of water is small that its depth d can be ignored

Using the relation $n_l = \frac{r}{IP}$ Where $r = 2f = 2 \times 8 = 16cm$

$\therefore \quad IP = \frac{r}{n_1} = \frac{16}{4/3} = 12cm$

Therefore the pin coincided with its image at a height of 12cm above the mirror

MEASUREMENT OF REFRACTIVE INDEX OF A GLASS BLOCK BY APPARENT DEPTH METHOD.



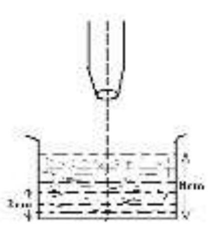
A vertically traveling microscope having a graduated scale T besides it is focused on lycopodium particles placed at O on a sheet of white paper. The scale reading a on T is noted. A glass block whose refractive index is required is placed on a paper and the microscope is raised until the particles are refocused at I. The scale reading b is again noted. Lycopodium particles are then poured on top of the glass block and the microscope is re-raised until the particles are again refocused. The new scale reading c is then noted. The refractive index of the block can then be calculated from $n = \frac{\text{Real depth}}{\text{Apparent depth}} = \frac{(c - a)}{(c - b)}$

NOTE

The refractive index of a liquid can be found by focusing on a sand particle at the bottom of an empty container and the scale reading is noted. The specimen liquid is then put in the container and the traveling microscope is refocused on the sand giving a scale reading b. Finally the traveling microscope is focused on the liquid surface giving a scale reading c. Thus $n_L = \frac{(c - a)}{(c - b)}$

EXAMPLE:

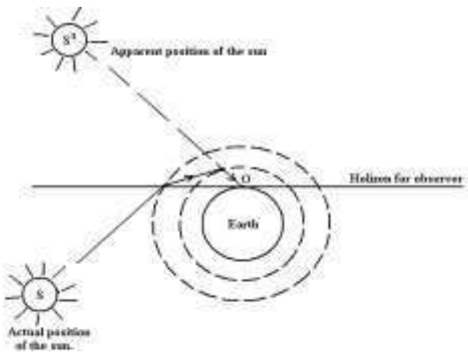
A microscope is focused on a mark at the bottom of the beaker. Water is poured in to the beaker to a depth of 8cm and it is found necessary to raise the microscope through a vertical distance of 2cm to bring the mark again in to focus. Find the refractive index of water.



Using the relation $n_g = \frac{\text{real depth}}{\text{apparent depth}} = \frac{8}{8 - 2}$

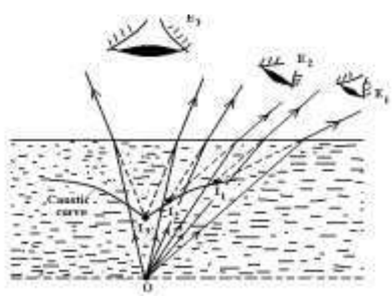
∴ $n_g = 1\cdot33$ (to 2 decimal places)

OBSERVATION OF SUNLIGHT SITUATED BELOW THE HORIZON.



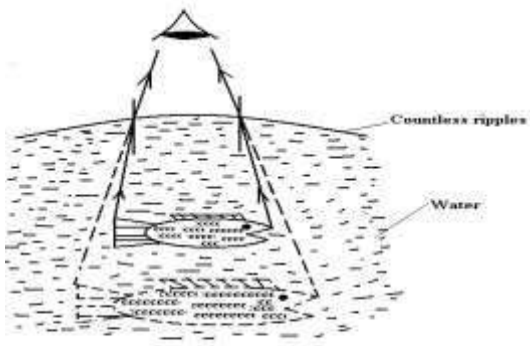
Light from the sun S is successively refracted from the different layers of the atmosphere due to the different optical densities they have. To an observer O on the earth's surface, sunlight appears to have come from point S^1.

THE APPARENT SHAPE OF THE BOTTOM OF A POOL OF WATER



When the observer moves the eye from E_1 towards E_3 the apparent position of O moves from I_1 to I_3 along a curve due to refracted rays bending away from the normal as shown above. Thus the bottom of the pool of water appears curved, being shallower near the edge than at the centre.

THE APARENT SIZE OF A FISH SITUATED IN WATER.



A large surface of water is not completely flat but consists of countless ripples whose convex surface on air acts as a convex lens of long focal length. In consequence the fish is within the focal length of the lens hence it appears magnified to an observer viewing it from above.

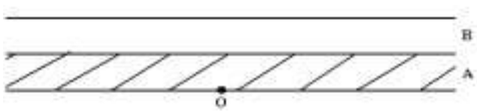
EXERCISE

1. Show that for an object viewed normally from above through a parallel sided glass block, the refractive index of the glass material is given by $n_g = \frac{\text{real depth}}{\text{Apparent depth}}$

2. Derive an expression for the apparent displacement of an object when viewed normally through a parallel sided glass block.

3. A vessel of depth 2d cm is half filled with a liquid of refractive index μ_1, and the upper half is occupied by a liquid of refractive index μ_2. Show that the apparent depth of the vessel, viewed perpendicularly is $\left(\frac{1}{\mu_1} + \frac{1}{\mu_2}\right) d$

4. Two parallel sided blocks A and B of thickness 4.0cm and 5.0cm respectively are arranged such that A lies on an object O as shown in the figure below



Calculate the apparent displacement of O when observed directly from above, if the refractive indices of A and B are 1.52 and 1.66.

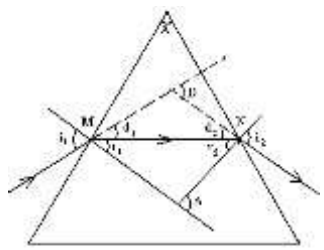
5. A tank contains liquid A of refractive index 1.4 to a depth of 7.0cm. Upon this floats 9.0cm of liquid B. If an object at the bottom of the tank appears to be 11.0cm below the top of liquid B when viewed directly above from, calculate the refractive index of liquid B.

6. Describe how the refractive index of a small quantity of a liquid can be determined using a concave mirror.
7. Describe how the refractive index of a glass block can be determined using the apparent depth method.

8. A small liquid quantity is poured into a concave mirror such that an object held above the liquid coincides with its image when it is at a height h from the pole of the mirror. If the radius of curvature of the mirror is r, show with the aid of a suitable illustration, that the refractive index of the liquid. $n = \frac{r}{h}$
9. Explain how light from the sun reaches the observer in the morning before the sun appears above the horizon
10. Explain the apparent shape of the bottom of a pool of water to an observer at the bank of the pool.
11. Explain why a fish appears bigger in water than its actual size when out of water.

DEVIATION OF LIGHT THRUOGH A PRISM

The angle of deviation caused by the prism is the angle between the incident ray and the emergent ray.
Consider a ray of light incident in air on a prism of refracting angle A and finally emerges into air as shown.



From the diagram above, MS and NS are normals at the points of incidence and emergence of the ray respectively.

 MPN + MSN = 180

Thus NST = MPN = A

Suppose i_1, r_1 and i_2, r_2 represents angles of incidence and refraction at faces M and N respectively, then

From geometry of MNS, $r_1 + r_2 = A$ ------------------ (i)

Total deviation $D = d_1 + d_2$ where $d_1 = i_1 - r_1$ and $d_2 = i_2 - r_2$

 $D = i_1 - r_1 + i_2 - r_2$

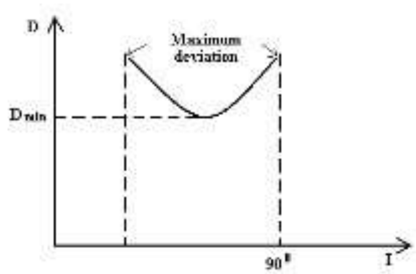
On simplifying, $D = i_1 + i_2 - (r_1+r_2)$ ------------- (ii)

Combining equation (i) and (ii) gives

$D = i_1 + i_2 - A$

NOTE:

Experiments show that as the angle of incidence i is increased from zero, the deviation D reduces continuously up to a minimum value of deviation D_{min} and then increases to a maximum value as the angle of incidence is increased as shown below:

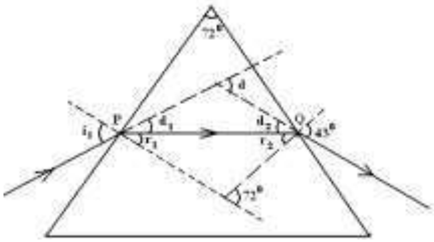


EXAMPLE:

1.A ray of light is incident on a prism of refracting angle 72□ and refractive index of 1□3. The ray emerges from the prism at 43□. Find

(i) the angle of incidence.

(ii) the deviation of the ray.



(i) At P, Snell's law becomes .

n_a sin 43□ = 1□3 sin r_2

□ r_2 = 31□64□

But $r_1 + r_2$ = 72□

□ r_1 = 72□ – r_2

= 72□ – 31□64□

□ r_1 = 40□36□

At Q, Snell's law becomes

n_a sin i_1 = 1□3 sin 40□36□

□ i_1 = 57□34□.

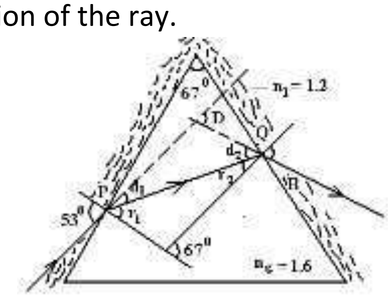
(ii) Total Deviation D = $d_2 + d_1$

where $d_1 = i_1$ □r_1 and $d_2 = i_2$ □ r_2

□ D = (43□ – 31□64□) + (57□34□ – 40□36□)

□ D = 28□34□.

2.A prism of refracting angle 67□ and refractive index of 1□6 is immersed in a liquid of refractive index 1□2. If a ray of light traveling through the liquid makes an angle of incidence of 53□ at the left face of the prism, Determine the total deviation of the ray.



Total Deviation D = $d_2 + d_1$

where $d_1 = i_1$ □r_1 and $d_2 = i_2$ □ r_2

□ D = (53□ □ r_1) □ (i_2 □r_2) ---------------------------------(i)

At P, Snell's becomes

1□2 sin 53□ = 1□6 sin r_1

□ r_1 = 36□8□

But r_1 + r_2 = 67□

□ r_2 = 67□ □ r_1

= 67□ □ 36□8□

□□ r_2 = 30□2□

At Q, Snell's law becomes

1□6 sin 30□2□ = 1□2 sin i_2

i_2 = 42□12□

Substituting for r_1, i_2, and r_2 in equation (i) gives

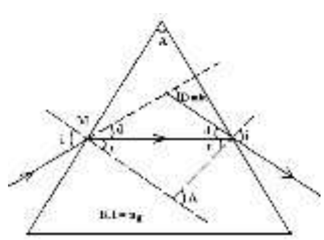
D = (53□ □ 36□8□) + (42□12□ □ 30□2□)

D = 28□12□

MINIMUM DEVIATION OF LIGHT BY A PRISM

.At minimum deviation, light passes symmetrically through the prism. That is to say, the angle of incidence is equal to the angle of emergence.

Consider a ray on one face of the prism at an angle $i_1$and leaves it at an angle i_2 to the normal as shown



For minimum deviation, i_1 = i_2 = i and r_1 = r_2 = r.

From the diagram, D_{min} = d + d

D_{min} = 2d where d = i □ r

□ D_{min} = 2i – 2r------------------------------(a) Moreover,

r + r = A.

□ 2r □ A **OR** r □ A⁄2 ------------------------(b)

Combining equation (a) and (b) gives.

D_{min} = 2i – A

□ i = $\frac{D_{min} + A}{2}$ --------------------------------(c)

At M Snell's law becomes

$n_a \sin i = n_g \sin r$

$\therefore \quad n_g = \dfrac{n_a \sin i}{\sin r}$ --------------------------------(d)

substituting equation (b) and (c) in (d) gives

$$n_g = n_a \frac{\sin\left(\frac{D_{min} + A}{2}\right)}{\sin \frac{A}{2}} \qquad \text{(e)}$$

Since $n_a = 1$,

$$\therefore \mathbf{n_g = \frac{\sin\left(\frac{D_{min} + A}{2}\right)}{\sin \frac{A}{2}}}$$

NOTE ;

Equation (e) suggests that if the prism was surrounded by a medium of refractive index n_l, then at minimum deviation

$$n_g = n_l \frac{\sin\left(\frac{D_{min} + A}{2}\right)}{\sin \frac{A}{2}}$$

EXAMPLES:

1. Calculate the angle of incidence at minimum deviation for light passing through a Prism of refracting angle 70° and refractive index of 1.65.

Solution

Using $\mathbf{n_g = n_a \dfrac{\sin\left(\frac{D_{min} + A}{2}\right)}{\sin \frac{A}{2}}}$ where $n_g = 1.65$, $A = 70°$ and $n_a = 1$

$$\therefore \quad 1.65 = \frac{1 \sin\left(\frac{D_{min} + 70°}{2}\right)}{\sin 70°/2}$$

On solving, $\mathbf{D_{min} = 72.33^\circ}$

The required angle of incidence $\mathbf{i = \frac{D_{min} + A}{2}} = \frac{72.33^\circ + 70^\circ}{2} = \mathbf{71.165^\circ}$

2.An equilateral glass prism of refractive index 1.5 is completely immersed in a liquid of refractive index 1.3. if a ray of light passes symmetrically through the prism, calculate the: (i) angle of deviation of the ray. (ii) angle of incidence ANALYSIS:

(a) For an equilateral prism, its refracting angle A =60°

(b) If the ray passes through the prism symmetrically, then the angle of deviation is minimum

(i) Using $\mathbf{\frac{n_g}{n_l} = \frac{\sin\left(\frac{D_{min} + A}{2}\right)}{\sin\frac{A}{2}}}$ where $n_g = 1.5$, $A = 60^\circ$ and $n_l = 1.3$

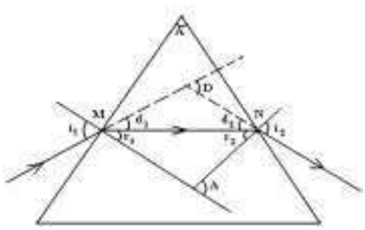
$$\frac{1.5}{1.3} = \frac{\sin\left(\frac{D_{min} + 60^\circ}{2}\right)}{\sin 60^\circ/2}$$

On solving, $\mathbf{D_{min} = 10.47^\circ}$

(ii) The required angle of incidence $\mathbf{i = \frac{D_{min} + A}{2}} = \frac{10.47^\circ + 60^\circ}{2} = \mathbf{35.235^\circ}$

DEVIATION OF LIGHT BY A SMALL ANGLE PRISM

The small refracting angles of this prism causes the angle i_1, r_1, r_2 and i_2 to be small such that $\sin i_1 \approx i_1$, $\sin r_1 \approx r_1$ $\sin r_2 \approx r_2$ and $\sin i_2 \approx i_2$.



From the diagram, $D = d_1 + d_2$

but $d_1 = i_1 - r_1$ and $d_2 = i_2 - r_2$

$\Rightarrow D = (i_1 - r_1) + (i_2 - r_2)$

On simplifying $D = i_1 + i_2 – (r_1 + r_2)$

but $r_1 + r_2 = A$

□ $D = i_1 + i_2 – A$----------------------(a) At

M Snell's law becomes.

$n_a \sin i_1 = n \sin r_1$

For small angles this gives $i_1 = nr_1$--------------------------------(b)

Similarly at N Snell's law becomes $i_2 = nr_2$-------------------------------(c)

Substituting equation (b) and (c) in (a) gives

$D = nr_1 + nr_2 – A$ D

$= n (r_1 + r_2) –A$

but $r_1 + r_2 = A$

□ $D = nA – A$

□ $D = (n – 1)A$

□ The deviation produced by a small angle prism is independent of the magnitude of the small angle of incidence on the prism. (ie: All rays entering a small-angle prism at small angles of incidence suffer the same deviation)

NOTE:

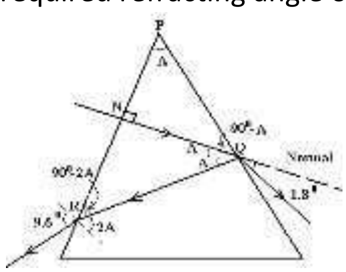
The result $D = (n – 1)A$ will later be used in developing lens theory.

EXAMPLES:

A ray of light that falls normally upon the first face of a glass prism of a small refracting angle under goes a partial refraction and reflection at the second face of the prism. The refracted ray is deviated through an angle 1□8□ and the reflected ray makes an angle of 9□6□ with the incident ray after emerging from the prism through its first face. Calculate the refracting angle of the prism and its refractive index of the glass material.

Solution

Let A be the required refracting angle of the prism as shown



Consider the deviation suffered by the incident light

$D = (n – 1) A$ where D = 1□8□

□ 1□8□ = $(n – 1) A$ -------------------------------------(i)

From □PQN, □PQN = 90□- A

□ At Q, the angle of incidence = A From

□NQR, □QRN = 90□- 2A

□ At R, the angle of incidence = 2A

□At R, Snell's becomes n_a sin 9□6□ = n sin 2A

For small angles, sin 9□6□ □ 9□6□ and sin A □ 2A

□ 9□6□ = 2nA --(ii)

Equation (i) □ Equation (ii) gives

$$\frac{1□8□}{9□6□} = \frac{(n-1)\,A}{2nA}$$

□ 3□6□n = 9□6□ (n – 1)

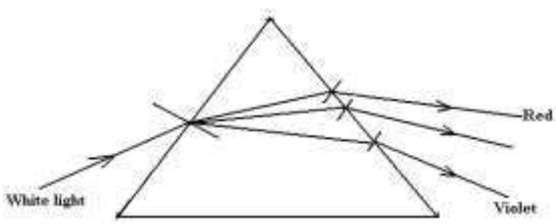
Thus n = 1□6

Equation (i) now becomes1□8□ = (1□6– 1) A

□ A = 3□

DISPERSION OF WHITE LIGHT BY A TRANSPARENT MEDIUM

Dispersion of whit light is the separation of white light in to its component colours by a transparent medium due to their speed differences in the medium.



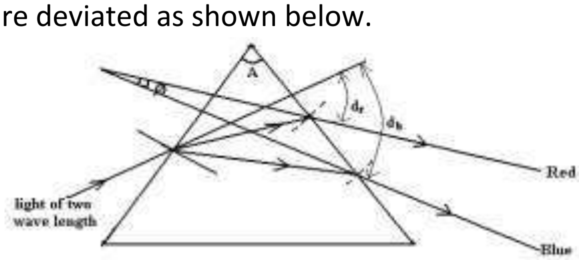
When white light falls on a transparent medium, its different component colours travel with different speeds through the medium. They are therefore deviated by different amounts on refraction at the surface of the medium and hence dispersion.

NOTE :

(i) White light is a mixture of various colours. This is called the spectrum of white light.

(ii) The spectrum of white light consists of red, orange, yellow, green, blue, indigo and violet light bands. On refraction, violet is the most refracted colour away from the normal (violet is the most deviated colour) while red is least deviated

(iii)When light of two wavelengths say red and blue light is incident at a small angle on a small angle prism of refracting angle A having refractive indices of n_r and n_b. for the two wave lengths respectively, then the two wave lengths are deviated as shown below.



The deviation of red and blue light is given by d_r = $(n_r – 1)A$

d_b = (n_b□ 1)A.

The quantity □ = d_b - d_r is called the Angular separation (Angular dispersion) produced by the prism.

□ □ = $(n_b – 1)$ A – $(n_r – 1)$ A

on simplifying □ = $(n_b – n_r)$ A

EXAMPLES:

1.Light of two wave length is incident at a small angle on a thin prism of refracting angle 5□ and refractive index of 1□52 and 1□48 for the two wave lengths. Find the angular separation of the two wave lengths after refraction by the prism.

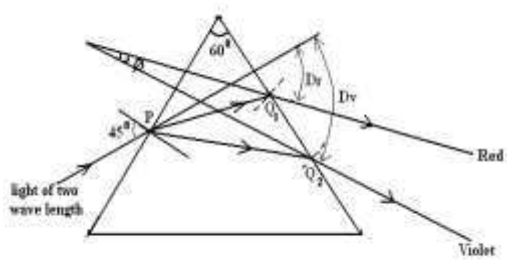
Solution

For a small prism, Angular separation □ = $(n_1 – n_2)$ A

□ = (1□52 –1□48) □5□

□ □ =0□2 □

2. A glass prism with refracting angle 60□ has a refractive index of 1□64 for red light and 1□66 for violet light. Calculate the angular separation of the red and violet rays which emerge from the prism when a ray of white light is incident on the prism at an angle of 45□



Case I:

Consider the deviation suffered by red light

At P, Snell's law becomes .

n_a sin 45□ = 1□64 sin r_1

□ r_1 = 25□54□

But $r_1 + r_2$ = 60□

□ r_2 = 60□ – r_1

= 60□ – 25□54□

□ r_2 = 34□46□

At Q_1, Snell's law becomes

n_a sin i_2 = 1□64 sin 34□46□

□ i_2 = 68□13□.

Total Deviation $D_r = d_2 + d_1$ where $d_1 = i_1$ □ r_1 and $d_2 = i_2$ □ r_2

□ D_r = (45□ – 25□54□) + (68□13□ – 34□46□)

□ D_r = 53□13□.

Case II:

Consider the deviation suffered by violet light

At P, Snell's law becomes .

n_a sin 45□ = 1□66 sin r_1

□ r_1 = 25□21□

But $r_1 + r_2$ = 60□

□ r_2 = 60□ – r_1

= 60□ – 25□21□

□ r_2 = 34□79□

At Q, Snell's law becomes

n_a sin i_2 = 1□66 sin 34□79□

□ i_2 = 71□28□.

Total Deviation $D_v = d_2 + d_1$ where $d_1 = i_1$ □r_1 and $d_2 = i_2$ □ r_2 □ D_v = (45□ – 25□21□) + (71□28□ – 34□79□) □ D_v = 56□28□.

Thus required angular separation □ = $D_v - D_r$

□ = 56□28□ – 53□13□

□ □ =3□15□

APPEARANCE OF WHITE LIGHT PLACED IN WATER

OBSERVATION:

A coloured spectrum is seen inside the water surface with violet on top and red down.

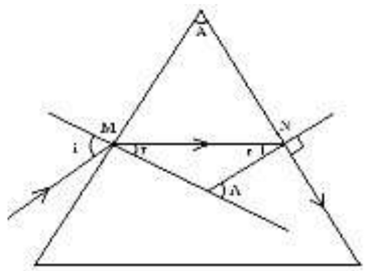
EXPLANATION:

The different component colours of white light travel with different speeds through water. They are therefore deviated by different amounts on refraction at the water surface. Hence different coloured images are formed at different points inside the water surface with a violet coloured image on top.

GRAZING PROPERTY OF LIGHT RAYS AS APPLIED TO PRISMS.

If a ray of light is either such that the incident angle or the emergent angle is equal to 90□ to the normal of the prism, then the ray is said to graze the refracting surface of the prism.

Consider a ray of light incident at an angle i on a glass prism of refracting angle A situated in air with the emergent light grazing the other refracting surface of the prism as shown.



From the diagram, r + c = A

$\therefore$ r = A – c----------------------------------(a)

At M Snell's law becomes

$n_a \sin i = n_g \sin r$----------------------------------(b)

Substituting equation (a) in (b) gives

$\sin i = n_g \sin (A - c)$

$\therefore \sin i = n_g (\sin A \cos C - \sin C \cos A)$-------------------(c)

At N, Snell's law becomes

$n_g \sin c = n_a \sin 90^\circ$.

$\therefore \sin c = \frac{1}{n_g}$

$$\therefore \cos c = \sqrt{1 - \sin^2 c} = \sqrt{1 - \frac{1}{n_g^2}} = \frac{\sqrt{n_g^2 - 1}}{n_g}$$

Substituting sin c and cos c in equation c gives

$$\sin i = n_g \left(\frac{\sqrt{n_g^2 - 1}}{n_g} \sin A - \frac{1}{n_g} \cos A \right)$$

On simplifying we have

$$\sin i + \cos A = \sin A \sqrt{n_g^2 - 1}$$

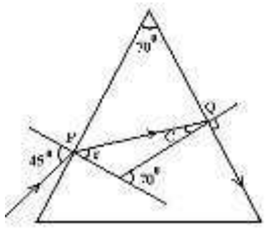
Squaring both sides and simplifying for n_g gives

$$\mathbf{n_g = \sqrt{1 + \left(\frac{\sin i + \cos A}{\sin A} \right)^2}}$$

Knowing the angles i and A, the refractive index n_g of a material of a prism can be determined.

EXAMPLES

1. Monochromatic light is incident at an angle of 45° on a glass prism of refracting angle 70° in air. The emergent light grazes the other refracting surface of the prism. Find the refractive index of the glass material.



At P, Snell's law becomes

$n_a \sin 45^\circ = n_g \sin r$ --------------------------------(a)

From the diagram, $r + c = 70^\circ$

$\Rightarrow \; r = 70^\circ - c$ -------------------------------(b)

Substituting equation (b) in (a) gives

$\sin 45^\circ = n_g \sin(70^\circ - c)$ -----------------------------------(c)

At Q, Snell's law becomes

$n_g \sin c = n_a \sin 90^\circ$

$$\Rightarrow n_g = \frac{1}{\sin c} \text{ ....(d)}$$

Substituting equation (d) in (c) gives

$$\sin 45^\circ = \frac{\sin(70^\circ - c)}{\sin c}$$

$\sin 45^\circ \sin c = \sin 70^\circ \cos c - \sin c \cos 70^\circ$

$\Rightarrow \; (\sin 45^\circ + \cos 70^\circ) \sin c = \sin 70^\circ \cos c$

Dividing cos c through out gives

$$\tan c = \frac{\sin 70^\circ}{\sin 45^\circ + \cos 70^\circ}$$

$$\Rightarrow c = 41.9^\circ$$

Equation (d) now becomes $n_g = \frac{}{\sin 41.9^o}$

$\therefore n_g = 1.497$

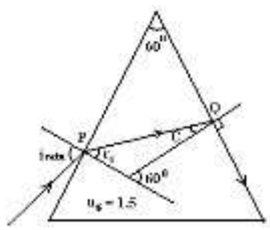
NOTE

For grazing condition, you may as well use the relation $n_g = \sqrt{1 + \left(\frac{\sin i + \cos A}{\sin A}\right)^2}$

$\therefore n_g = \sqrt{1 + \left(\frac{\sin 45^o + \cos 70^o}{\sin 70^o}\right)^2}$

$\therefore n_g = 1.498$

2..A ray of light is incident on one refracting face of a prism of refractive index 1.5 and refracting angle 60°. Calculate the minimum angle of incidence for the ray to emerge through the second refracting face.



ANALYSIS

for minimum angle of incidence, the emergent ray grazes the second refracting face.

At Q, Snell's law becomes

$1.5 \sin c = n_a \sin 90^o$

$\therefore c = 41.8^o$

But $r + c = 60^o$

$\therefore r = 60^o - c$

$= 60^o - 41.8^o$

$\therefore r = 18.2^o$

At P, Snell's law becomes

$1.5 \sin 18.2^o = n_a \sin i_{min}$

$\therefore i_{min} = 27.9^o$

NOTE

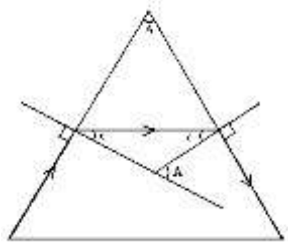
For grazing condition, you may as well use the relation $n_g = \sqrt{1 + \left(\frac{\sin i + \cos A}{\sin A}\right)^2}$

$$1.5 = \sqrt{1 + \left(\frac{\sin i_{min} + \sin 60 \cos 60^{o}}{\sin 60^{o}}\right)^{2}}$$

On simplifying, $i_{min} = 27.9^{o}$

LIMITING ANGLE OF THE PRISM

This is the maximum refracting angle of the prism for which he emergent ray grazes the second refracting surface. Suppose the incident ray grazes the first refracting surface then the limiting angle A is given by A = 2c where c is the critical angle of the glass air interface as shown.



EXERCISE:

1. (i) Obtain an expression relating the deviation of a ray of light by the prism to the refracting angle and the angles of incidence and emergence.

(ii) The deviation of a ray of light incident on the first face of a 60° glass prism at an angle of 45° is 40°. Calculate the angle of emergence of a ray on the second face of the prism (Ans $i_2 = 65^{o}$)

(iii) A prism of refractive index 1.64 is immersed in a liquid of refractive index 1.4. A ray of light is incident on one face of the prism at an angle of 40°. If the ray emerges at an angle of 29°, determine the angle of the prism.

(Answer: 57.7°)

2. (i) For a ray of light passing through the prism, what is the condition for minimum deviation to occur?

(ii) Derive an expression for the refractive index of a prism in terms of the refracting angle, A, and the angle of minimum deviation D.

(iii) A glass prism of refractive index n and refracting angle, A, is completely immersed in a liquid of refractive index n_i. If a ray of light that passes symmetrically through the prism is deviated through an angle φ, Show that

$$\frac{n_i}{n} = \frac{\sin \frac{A}{2}}{\sin\left(\frac{\varphi + A}{2}\right)}$$

(2)

3.(a) A glass prism with refracting angle 60° is made of glass whose refractive indices for red and violet light are respectively 1.514 and 1.530. A ray of white light is set incident on the prism to give a minimum deviation for red light. Determine the:

(i) angle of incidence of the light on the prism.

(ii) angle of emergence of the violet light.

(iii) angular width of the spectrum.

(b) A certain prism is found to produce a minimum deviation of 51°. While it produces a deviation of 62.8° for a ray of light incident on its first face at an angle of 40.1° and emerges through its second face at an angle of 82.7°. Determine the:

(i) refracting angle of the prism.

(ii) angle of incidence at minimum deviation.

(iii) refractive index of the material of the prism.

[Ans (i) 60° (ii) 55.5° (iii) 1.648]

4. (i) A ray of monochromatic light is incident at a small angle of incidence on a small angle prism in air. Obtain the expression D = (n – 1)A for the deviation of light by the prism.

(ii) A glass prism of small angle, A, and refractive index n_g and is completely immersed in a liquid of refractive index n_L. Show that a ray of light passing through the prism at a small angle of incidence suffers a deviation given by

$$D = \left(\frac{n_g}{n_L} - 1 \right) A$$

5. Explain why white light is dispersed by a transparent medium.

6. Light of two wave length is incident at a small angle on a thin prism of refracting angle 5° and refractive index of 1.52 and 1.48 for the two wave lengths. Find the angular separation of the two wave lengths after refraction by the prism.

[Ans δ = 0.2°]

7.A point source of white light is placed at the bottom of a water tank in a dark room. The light from the source is observed obliquely at the water surface. Explain what is observed.

8. Monochromatic light is incident at an angle i on a glass prism of refracting angle, A, situated in air. If the emergent light grazes the other refracting surface of the prism, Show that the refractive index, n_g, of the prism material is given by

$$n_g = \sqrt{1 + \left(\frac{\sin i + \cos A}{\sin A} \right)^2}$$

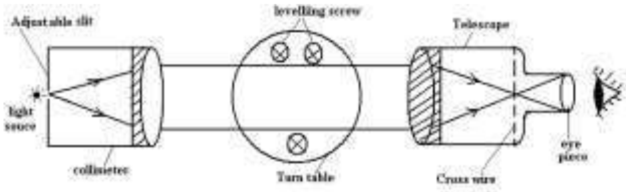
9. A ray of light is incident at angle of 30□ on a prism of refractive index 1□5 .calculate the limiting angle of the prism such that the ray does not emerge when it meets the second face.

□ Ans A = 61□3□ □

A SPECTROMETER

It is an instrument used to measure accurate determination of deviation of a parallel beam of light which has passed through a prism. This provides a mean of studying optical spectra and measurement of refractive indices of glass in form of a prism.

It consists of a collimator, a telescope, and a turn table on which the prism is placed as shown.



Before the spectrometer is put in to use, 3 adjustments must be made onto it and these include,

(i) The collimator is adjusted to produce parallel rays of light.

(ii) The turntable is leveled.

(iii)The telescope is adjusted to receive light from the collimator on its crosswire.

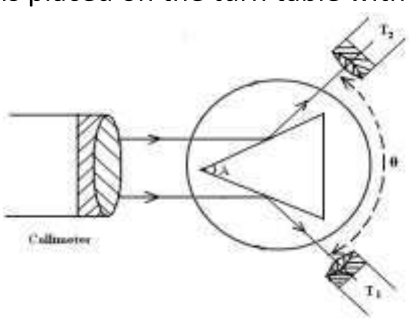
MEASUREMENT OF THE REFRACTING ANGLE "A" OF THE PRISM

The collimator is adjusted to produce parallel rays of light.

The turn table is leveled.

The telescope is adjusted to receive light from the collimator on its cross wire.

The prism is placed on the turn table with its refracting angle facing the collimator as shown.

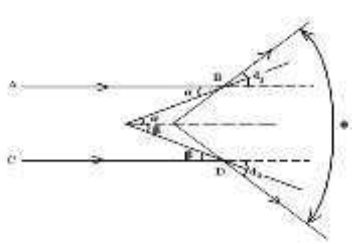


With the table fixed, the telescope is moved to position T_1 to receive the light from the collimator on its cross wire. This position T_1 is noted and the telescope is turned n to a new position T_2 to receive light on its cross wire. The angle □ between T_1 and T_2

Is measured. The prism angle A is given by A = $\frac{1}{2}$ □

PROOF OF THE RELATION

Consider a parallel beam of light incident on to a prism of refracting angle A making glancing angles □ and □ as shown.



From the geometry, □ + □ = A------------------------------
.(i). Deviation d_1 of ray AB = 2□
Deviation d_2 of ray CD = 2□.

Total deviation □ = $d_1 + d_2$

= 2□ + 2□

= 2 (□ + □)--------------------------(ii)

Combining equation (i) and (ii) gives

□ = 2A.

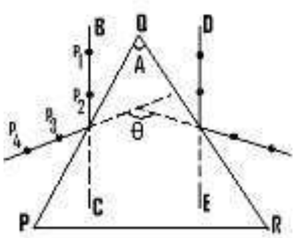
NOTE :

It is now clear from the geometry that the angle □ turned through in moving the telescope from T_1 to T_2 is given by □ = 2A

Thus A = $\frac{1}{2}$□.

METHOD 2: USING OPTICAL PINS

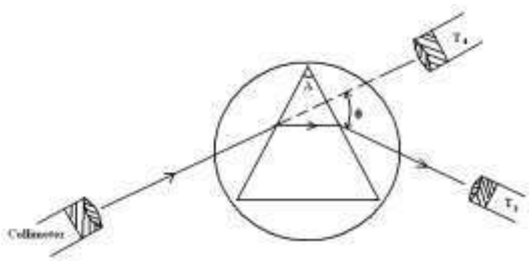
A white paper is stuck to the soft board using top-headed pins. Two parallel line are AB and DC are drawn on the paper and the prism is placed with its apex as shown.



Two optical pins $P_1$and P_2 are placed along AB and pins P_3 and P_4 are placed such that they appear to be in line with the images of $P_1$and P_2 as seen by reflection from face PQ .The procedure is repeated for face QR. The prism is removed and angle □ is measured.The required refracting angle **A** □ $\frac{\theta}{2}$

MEASUREMENT OF MINIMUM DEVIATION “D_{min}” OF THE PRISM

The collimator is adjusted to produce parallel rays of light.
The turn table is leveled.
The telescope is adjusted to receive light from the collimator on its cross wire
The prism is placed on a turn table with its refracting angle facing away from the collimator as shown.



The telescope is turned in the direction of the base of the prism until light can be seen. With light kept in view, both the telescope and the table are turned until light moves in the opposite direction. Position T_3 of the telescope is noted.

The table is then fixed and the prism is removed so that the telescope is turned to a new position T_4 to receive the un deviated light. The angle between T_3 and T_4 is determined and this is the angle of minimum deviation D_{min}.

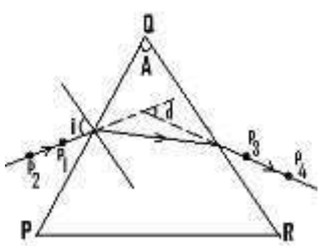
NOTE:

(i) Position T_3 is noted because, in the position of minimum deviation light viewed through the telescope moves in the opposite direction.

(ii)The refracting index of a glass prism of known refracting angle A can be determined using a spectrometer from the relation

$$n = \frac{\sin \frac{(D_{min} + A)}{2}}{\sin {}^{A}\!/_{2}}$$

METHOD 2: USING OPTICAL PINS



Two optical pins P_1 and P_2 are placed along the lines that make different angles of incidence i. Pins P_3 and P_4 are placed such that they appear to be in line with the images of P_1 and P_2 as seen through the prism. The angles of deviation d are measured for different angles of incidence. A graph of d against i is plotted to give a curve whose angle of deviation at its turning point is the angle of minimum deviation D_{min} of the prism.

USES OF A GLASS PRISM.

1. They enable the refractive index of a glass material to be measured accurately.
2. They are used in the dispersion of light emitted by glowing objects.
3. They are used as reflecting surfaces with minimal energy loss.

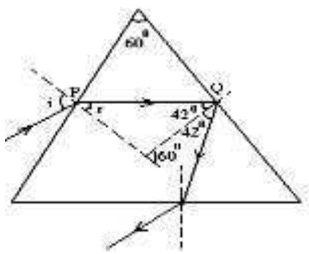
4. They are used in prism binoculars.

More worked out examples

1.A ray of monochromatic light is incident on one face of a glass prism of refracting angle 60□ and is totally internally reflected at the next face.

(i) Draw a diagram to show the path of light through the prism.

(ii) Calculate the angle of incidence at the first face of the prism if its refractive index is 1□53 and the angle of incidence at the second face is 42□.



From the diagram, r + 42□ = 60□

□ r = 18□

At P, Snell's becomes

n_a sin i = 1□53 sin 18□

□ □ i = 28□2□

NOTE

For n_g =1□53, then the critical angle c for the above glass material is given by the relation

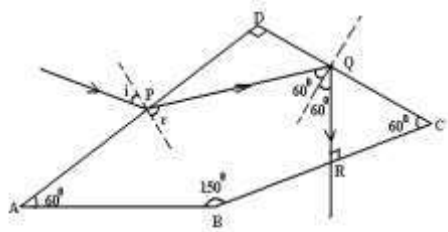
$\sin c$ □ $\frac{1}{n_g}$ □ $\frac{1}{1□53}$

□ c □ 40□8°

Thus total internal reflection occurs at Q since the angle of incidence is greater than the critical angle c

2. A ray of light is incident on the face AD of a glass block of refractive index 1□52 as shown.

If the ray emerges normally through face BC after total internal reflection, Calculate the angle of incidence, i.



ANALYSIS

(i) Its after a total internal reflection at Q that the ray emerges through face DC

(ii) At R, there is no refraction. There fore Snell's law does not hold at this point.

From ∆RCQ, ∠RQC + 60° + 90° = 180°

∴ ∠RQC = 30°

∴ At Q, the angle of reflection = 60°

Hence at Q, the angle of incidence = 60°

Solving ∆QDP gives ∠QPD = 60°

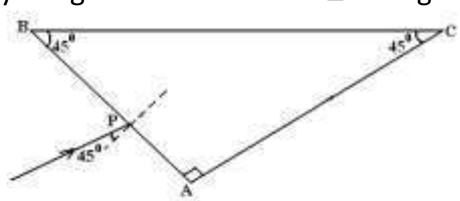
Hence at P, the angle of refraction r = 30°

∴ At P, Snell's law becomes

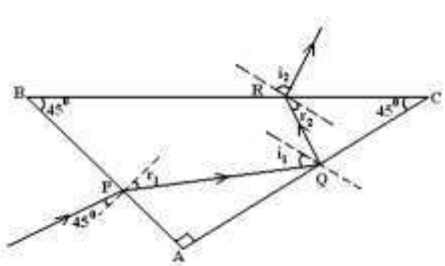
$n_a \sin i = 1{\cdot}52 \sin 30°$

∴ $i = 49{\cdot}5°$

3. A ray of light is incident at 45° on a glass prism of refractive index 1·5 as shown.



Calculate the angle of emergence and sketch the ray diagram.



Solution

At P, Snell's becomes $n_a \sin 45° = 1{\cdot}5 \sin r$

∴ ∴ $r_1 = 28{\cdot}1°$

At P, $\angle APQ + r_1 = 90°$ where $r_1 = 28{\cdot}1°$

∴ $\angle APQ = 61{\cdot}9°$

From ΔAPQ, $\angle PQA + 61.9^\circ + 90^\circ = 180^\circ$

$\therefore \Rightarrow \angle PQA = 28.1^\circ$

$\therefore$ At Q, the angle of incidence $i = 61.9^\circ$

Tesiting for total internal reflection at Q using the relation $\sin c = \frac{1}{n_g} = \frac{1}{1.5}$ **gives** $c = 41.8^\circ$

Thus light is totally reflected at Q since $i > c$.

$\therefore \angle PQA = \angle RQC = 28.1^\circ$

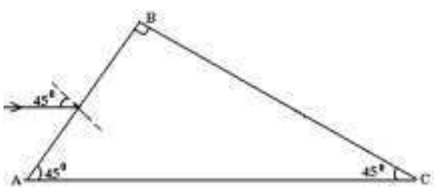
From ΔRQC, $28.1^\circ + 45^\circ + 90^\circ + r_2 = 180^\circ$

$r_2 = 16.9^\circ$

At R, Snell's becomes $1.5 \sin 16.9^\circ = n_a \sin i_2$

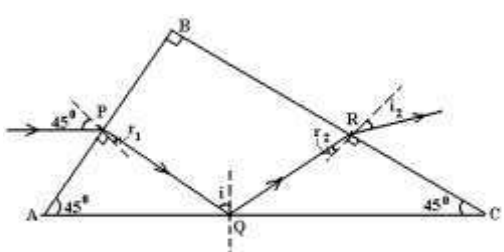
Thus $i_2 = 25.15^\circ$

4. A ray of light is incident at 45° on a glass prism of refractive index 1.5 as shown.



Calculate the angle of emergence and sketch the ray diagram.

Solution



At P, Snell's becomes $n_a \sin 45^\circ = 1.5 \sin r$

$\therefore \Rightarrow r_1 = 28.1^\circ$

From ΔAPQ, $\angle PQA + 45^\circ + 90^\circ + r_1 = 180^\circ$ where $r_1 = 28.1^\circ$ $\therefore \Rightarrow \angle PQA = 16.9^\circ$

$\therefore$ At Q, the angle of incidence $i = 73.1^\circ$

1 1

Tesiting for total internal reflection at Q using the relation $\sin c = \frac{1}{n_g} = \frac{1}{1.5}$ **gives** $c = 41.8°$

Thus light is totally reflected at Q since i > c.

$\therefore \angle PQA = \angle RQC = 16.9°$

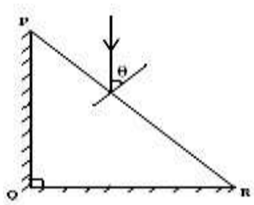
From $\angle RQC$, $16.9° + 45° + 90° + r_2 = 180°$

$r_2 = 28.1°$

At R, Snell's becomes $1.5 \sin 28.1° = n_a \sin i_2$

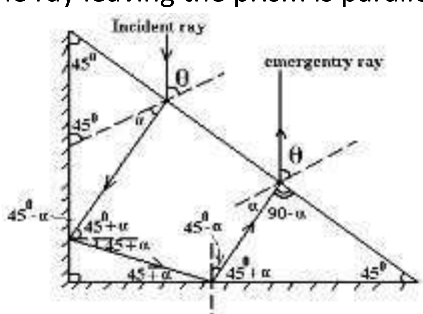
Thus $i_2 = 45°$

5.The diagram in the figure below shows a cross section of an isosceles right angled prism sides PQ and QR are coated with a reflecting substance. A ray of light is incident on PR at an angle θ as shown



(i) Draw a diagram to show the path of light through the prism.

(ii) Show that the ray leaving the prism is parallel to the incident ray.



From geometry of the figure above the angle of emergence at D is the same as the incident angle θ at A. Hence the emergent ray is parallel to the incident ray.

EXERCISE

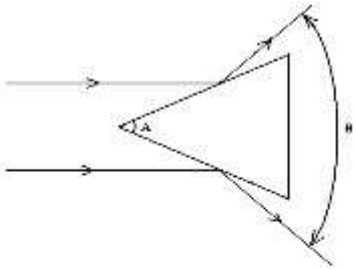
1. Draw a labeled diagram of a spectrometer and State the necessary adjustments that must be made on to it before put in to use.

2. Describe how the refracting angle of the prism can be measured using a spectrometer.

3.You are provided with pins, a white sheet of paper, a drawing board and a triangular

prism. Describe how you would determine the refracting angle A of the prism

1. A parallel beam of light is incident on to a prism of refracting angle, A, as shown



Show that □ = 2A

5.Describe how the minimum deviation, D, of a ray of light passing through a glass prism can be measured using a spectrometer.

6.You are provided with pins, a white sheet of paper, a drawing board and a triangular prism. Describe how you would determine the angle of minimum deviation, D, of a ray of light passing through a glass prism.

7. Describe how the refractive index of a material of a glass prism of known refracting angle can be determined using a spectrometer. 8. Describe briefly two uses of glass prisms