RESOURCE MOCK EXAMINATION 2016

P425/1
PURE MATHEMATICS
PAPER 1
3 HRS

INSTRUCTIONS:
Answer all questions in Section A, and only **five** questions from Section B

SECTION A (40 MARKS)

1. Solve the equation: $5^{\log_{25} x} = 3^{\log_{27} 2x}$ (5 Marks)

2. Calculate the perpendicular distance between the parallel lines
$3x + 4y + 10 = 0$ *and* $3x + 4y - 15 = 0$. (05 Mks)

3. Solve the equation: $\cos(\theta + 60^0) = 1 + \cos\theta$ *for* $-180^0 < \theta < 360^0$. (05 Mks)

4. Find and sketch the locus: $Arg(iZ + 1) = {}^{\pi}/_{4}$, where $Z = x + iy$. (05 Mks)

5. Given that $ye^x = \sin x + \cos x$, show that $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$. (05 Mks)

6. The line $\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ a \end{pmatrix} + \lambda \begin{pmatrix} b \\ 1 \\ -2 \end{pmatrix}$ is parallel to the plane $x + 2y + 2Z = 2$, and the perpendicular distance of the line from the plane is +3 units. Find the values of a and b. (05 Mks)

7. Find $\int cosec\theta \sec\theta d\theta$ (05 Mks)

8. Solve the differential equation: $x\frac{dy}{dx} + y = xy$ *for* $y = 1$ when $x = 1$. (05 Mks)

SECTION B (60 MARKS)

9. a) Use the binomial theorem to show that;
$\left(\sqrt{1 + 2x} + \sqrt{1 - 4x}\right)^2 - x - {}^{5}/_{2}x^2 + \cdots$ (07 Mks)

b) Taking $x = {}^{1}/_{16}$ use the expansion in (a) above to estimate $\sqrt{6}$ to 2 decimal places. (05Mks)

10. a) Solve the equation $cos\theta + sin2\theta = 0$ for $0 < \theta < 2\pi$ (05 Mks)

b) Show that in any triangle ABC, $cos\frac{1}{2}(A - B) = \frac{a+b}{c}sin\frac{C}{2}$, hence find the interior angles of the triangle if C = 60^0, a:c = 2:5 and b:c = 1:2 (07 Mks)

11. a) The area of a circle increases at a rate of $40\pi cm^2S^{-1}$ at the instant when its circumference is $10\pi cm$. Fin the rate at which the circumference increases at this instant. (06 Mks)

b) Use small changes to evaluate $tan44^0 48^1$ to 4 decimal places. (06 Mks)

12. a) A and B are the points (3,1,1) and (5,2,3) respectively, and C is a point on the line $r = \begin{pmatrix} 2 \\ 4 \\ -2 \end{pmatrix} + \lambda\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$. If angle BAC=$90^0$, find the coordinates of C (06 Mks)

b) The vector $\begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix}$ is perpendicular to the plane containing the line $\frac{x-3}{-2} = \frac{y+1}{a} = \frac{2-2}{1}$,

find the; i) value of a

ii) cartesian equation of the plane. (06 Mks)

13. a) Solve the equation: $2Z - i\bar{Z} = 5 - i$, where $Z = x + iy$. (06 Mks)

b) Find, in Cartesian form, the cube roots of $-\sqrt{2} + i\sqrt{6}$. (06 Mks)

14. A variable line through the point C(3,4) meets the positive axes in the points P and Q. O is the origin. Given that P is the point (t,o)

a) Show that the area of ΔOPQ is $A = \frac{2t^2}{t-3}$

b) Find the minimum value of A. (12 Mks)

15. Find a) $\int \frac{dx}{\sqrt{(1-cos2x)}}$ (05 Mks)

b) $\int \frac{dx}{x^2(x^2-1)}$ (07 Mks)

16. The differential equation $\frac{dx}{dt} = \frac{-1}{20}x$ represents the rate at which a radioactive substance disintegrates.

a) Given that an amount x_0 takes 20 minutes to reduce to $^1/_3\, x_0$, calculate how much longer it takes an amount $2x_0$ to reduce to $^1/_3\, x_0$.

b) Find the mass of the substance that reduces to 15 grammes in 15 minutes. (12 Mks)

END